Math 2001, Spring 2023. Katherine E. Stange.

1 Assignment

Prove that the set $\{2x + y : x \in \mathbb{Z}, y \in \{1, 0\}\}$ is the set of all integers.

Hints: You might find Example 1.2 is helpful for inspiration, but don't try to copy the format. Write it the way you feel is best. Also, it may be helpful to start writing out examples of things in the set. For example, taking $x = 1, y = 1$ *gives* $2x + y = 3$*, so* 3 *is in the set.*

2 A possible solution

Let us set the notation $S = \{2x + y : x \in \mathbb{Z}, y \in \{1, 0\}\}$.

Since x and y are integers, $2x + y$ is an integer. Therefore every element of S is an integer.

On the other hand, suppose we are given an integer n and we wish to show that $n \in S$. If n is even, then $n = 2x$ for some integer x, so n is of the form $2x + y$ where $y = 0$. Therefore $n \in S$. If n is odd, then $n = 2x + 1$ for some integer x, so n is of the form $2x + y$ where $y = 1$. Therefore $n \in S$.

Since all elements of S are integers and every integer is an element of S, we have $S = \mathbb{Z}$.

3 Another possible solution

Write $S = \{2x + y : x \in \mathbb{Z}, y \in \{1, 0\}\}$. Since it is clear that $S \subseteq \mathbb{Z}$, we need only show that $\mathbb{Z} \subseteq S$. Let $s \in \mathbb{Z}$. We will show that $s \in S$.

If s is even, we can write $s = 2n$ for some n. Thus $s = 2x + y$ where $x = n$ and $y = 0$. Thus $s \in S$.

If s is odd, we can write $s = 2n + 1$ for some n. Thus $s = 2x + y$ where $x = n$ and $y = 1$. Thus $s \in S$.

Since any integer is either even or odd, we have shown that $\mathbb{Z} \subseteq S$.

4 ChatGPT

Me: Consider the set $\{2x + y : x \in \mathbb{Z}, y \in \{0, 1\}\}$. Show that this set is equal to the set of the integers.

ChatGPT: The set $\{2x + y : x \in \mathbb{Z}, y \in \{0, 1\}\}$ includes all integers since any integer can be expressed as $2x + y$ for some x in $\mathbb{Z}$ and y in $\{0, 1\}$. For example, the integer 4 can be expressed as $2(2) + 0$ and the integer 5 can be expressed as $2(2) + 1$. Therefore, the set $\{2x + y : x \in \mathbb{Z}, y \in \{0, 1\}\}$ is equal to the set of the integers.

Me: Can you give a more detailed proof?

ChatGPT: Sure.

First, let's assume that the set $\{2x + y : x \in \mathbb{Z}, y \in \{0, 1\}\}$ is a subset of the set of integers. To show that it is actually equal to the set of integers, we need to prove that every integer is also in the set $\{2x+y : x \in \mathbb{Z}, y \in \{0, 1\}\}$.

To do this, we can consider an arbitrary integer n. We can express n as $2x + y$ for some x in $\mathbb{Z}$ and y in $\{0, 1\}$ by dividing n by 2 and noting the remainder.

For example, if n is even, then $n = 2x$ for some x in $\mathbb{Z}$. This means that $y = 0$ and n can be expressed as $2x + 0$.

If n is odd, then $n = 2x + 1$ for some x in $\mathbb{Z}$. This means that $y = 1$ and n can be expressed as $2x + 1$.

Since any integer can be expressed as $2x + y$ for some x in $\mathbb{Z}$ and y in $\{0, 1\}$, it follows that the set $\{2x + y : x \in \mathbb{Z}, y \in \{0, 1\}\}$ is equal to the set of integers.