

# The Sandbox puzzle

Katherine Stange, Math 2001, CU Boulder

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5. You begin with zero sand, and can remove or add sand by the 6 or 15-gallon pailful.
6. **What amounts of sand are possible to obtain?**

## Some thoughts from the class discussion

1. We can get 0 by not doing anything.
2. We can get 6 by adding one 6-gallon scoop.
3. We can get 9 by adding a 15-gallon scoop and taking away a 6-gallon scoop ( $12 = 15 - 6$ ).
4. We can get 3 by adding a 15-gallon scoop and taking away two 6-gallon scoops ( $3 = 15 - 2 \cdot 6$ ).
5. We can get  $-6$  by taking away a 6 gallon scoop.
6. It looks like we can get anything that 'looks like'  $15x + 6y$ .



# Your task is to prove this theorem

## Theorem

*In the sandbox puzzle, the amount of sand that are possible to obtain are exactly those which are a multiple of 3 gallons (positive, zero or negative).*

Important: This actually says two things:

1. We can get every multiple of 3.
2. Anything we get is a multiple of 3.

So the proof may need to prove these two things separately.