Worksheet on Quantifiers
Katherine E. Stange, Math 2001

1 Quantifiers and Negation

For each english sentence, write it in symbols: **no english words!** You can use such things as quantifiers ($\exists$, $\forall$), boolean operators ($\sim$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$), etc.

x is unquantified

1. The set X has 3 elements.

   $|X| = 3$

2. The number x is odd.

   $x = 2k+1,\ k \in \mathbb{Z}$

   $\exists k \in \mathbb{Z},\ x = 2k+1.$

"$\forall X, |X| = 3$"

"$\forall x, \exists k, x = 2k+1$"

All x are odd.

x is universally quantified

3. Every rational number is an integer.

   $\forall x \in \mathbb{Q},\ x \in \mathbb{Z}.$

$\mathbb{Q} \subseteq \mathbb{Z}.$ ✓

$$\boxed{\begin{array}{c}\text{Every rational is a ratio of integers.}\\ \forall x \in \mathbb{Q},\ \underline{\exists a, b \in \mathbb{Z},\ x = \tfrac{a}{b}}.\end{array}}$$

logically equivalent

4. Every integer is odd.

   $\forall x \in \mathbb{Z},\ \underbrace{\exists k \in \mathbb{Z},\ x = 2k+1}_{\text{"}x \text{ is odd"}}.$

5. If $x \in \mathbb{Z}$, then x is odd.

   $x \in \mathbb{Z} \Rightarrow (\exists k \in \mathbb{Z},\ x = 2k+1)$

$P \Rightarrow Q$

same as

$\forall x,\ P(x) \Rightarrow Q(x)$

$\forall\ \underline{x \in \{x : P(x)\}},\ \underline{Q(x)}$

6. If $x \in \mathbb{R}$, then $x \in \mathbb{Q}$.

7. There exists an odd integer.

8. There exists a subset of the integers of cardinality 3.

9. If X has 3 elements, then there exists a subset of X having 2 elements.

10. For every integer, there's an additive inverse (something which adds to it to give 0).