Worksheet on Sets, Quantifiers, Negation, Example/Counterexample
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1 Quantifiers and Negation

1. Negate the following statements. Which is true, the original or the negation? If an existential statement is true, demonstrate it with an example. If a universal statement is false, demonstrate it with a counterexample.

   (a) $\forall x \in \mathbb{Z}, x > 3$.

   (b) $\forall x \in \mathbb{Z}, 0x = 0$.

   (c) $\exists x \in \mathbb{Z}, x$ is prime and negative.

   (d) $\exists x \in \mathbb{Z}, x = -x$.

2 Sets and Quantifiers

In this section, consider the set $S = \{1, 2, \{2\}, 3, \{1, 2\}, \{4\}\}$.

1. For each statement, agree on what it means, and determine if it is true or false. If it is a false universal statement, give a counterexample. If it is a true existential statement, give the example that makes it true. To make this more fun, circling the letter under the correct column for each statement will spell something.

	TRUE	FALSE
$\exists x \in S, x = 2$	S	G
$\exists x \in S, x = 4$.	A	E
$\forall x \in S, x = 2$.	R	T
$\forall x \in S, x \in \mathbb{Z}$.	T	S
$\forall x \in S, x \subseteq \mathbb{Z}$.	C	A
$\exists x \in \mathbb{Z}, x \in S$.	R	L
$\exists x \in S, x \in \mathbb{Z}$.	E	A
$\exists x \in S, \{x\} \in S$.	T	S
$\forall x \in S, (x \in \mathbb{Z}) \vee (x \subseteq \mathbb{Z})$.	H	O
$\forall x \in \mathbb{Z}, x \notin S$.	O	E
$\forall x \in \mathbb{Z}, \{x\} \notin S$.	D	C
$\forall x \in \mathbb{Z}, \{\{x\}\} \notin S$.	A	O
$\forall x \in S, \{x\} \subseteq S$.	T	G
$\forall x \in \mathbb{Z}, (1 \leq x \leq 4 \implies x \in S)$.	H	S
$\forall x \in \mathbb{Z}, (1 \leq x \leq 3 \implies x \in S)$.	P	D
$\forall x \in \mathbb{Z}, (x \in S \implies \{x\} \in S)$.	A	J
$\forall x \in \mathbb{Z}, (x \in S \implies \{x\} \subseteq S)$.	S	W

2. In the long list of statements about S of the previous problem, identify those which are logical negations of one another. For each such pair, one should be true and one should be false. Did you make any errors?