

## MATHEMATICS 2001 – EXISTENCE PROOF PUZZLES!

**It's puzzle time!** The goal today is to produce some nicely-written proofs.

**Theorem 1.** *There exists a set of any finite cardinality.*

**Theorem 2.** *Every odd integer is the sum of two consecutive integers.*

**Theorem 3.** *Every odd integer is the difference between two consecutive perfect squares.*

**Theorem 4.** *There are arbitrarily large gaps in the sequence of prime numbers.*

**Theorem 5.** *Between any two irrational numbers, there is a rational number.*