

Proof Quiz #5 Solution

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Honor Code Rules

Proof Quizzes are open book, but are to be completed on your own without collaboration. To be specific, you may use your course notes, textbook, course website resources, course videos. You may not use the internet beyond the course websites. You may not ask anyone else for help (except your professor), including other humans, or posting/entering your question into the internet. You may not share the questions or answers with anyone else.

Have you read, understood, and followed the honor code rules above?

YES / NO

Please write your best written proof of the following theorem. You will be graded on logic as well as writing.

Theorem 1. *Suppose $x \in \mathbb{R}$. If $x(x+1)(x+2) \leq 0$ or $x(x+1) \leq 0$, then $x \leq 0$.*

Hint: Use contrapositive proof. And don't forget that $\sim(P \vee Q) = (\sim P) \wedge (\sim Q)$ (i.e. be careful with your negation of an "OR"!).

Solution

Note: The correct contrapositive statement is the following:

Theorem 2. *Suppose $x \in \mathbb{R}$. If $x > 0$, then $x(x+1)(x+2) > 0$ and $x(x+1) > 0$.*

Notice the 'and' in the conclusion.

In the example solution below, I have not explicitly stated the contrapositive, but you can see how I have used it.

Proof. Suppose that $x \in \mathbb{R}$. For a proof by contrapositive, suppose that $x > 0$. Then $x+1 > 0$ and $x+2 > 0$. Since a product of positive real numbers is positive, we have that $x(x+1) > 0$ and $x(x+1)(x+2) > 0$. \square