

Quiz Proof #3

February 9, 2018

Tools

You may use the following definition.

Definition 1. A real number x is rational if it can be expressed as $x = \frac{a}{b}$ where $a, b \in \mathbb{Z}$. Otherwise, it is irrational.

You may use the following theorem as a known fact.

Lemma 1. The real number $\sqrt{2}$ is irrational.

You may use any basic facts about rationals (e.g. product of rationals is rational, integers are rational).

You may **not** use as a known fact that the product of a rational and an irrational is irrational (this would make the proof too short). Similarly, other statements combining rationals and irrationals (sums/products/differences/quotients) are not allowed. You are welcome, however, to prove these facts or special cases of them, as part of your proof.

Hint/Note: My intention is for the proof to be fairly short (using the lemma above), but not totally trivial. You don't need to mimic the proof that $\sqrt{2}$ is irrational, which is more work. Use proof by contradiction and make use of the lemma to save time compared to that approach.

Task

Please prove the following theorem.

Theorem 1. The real number $7\sqrt{2}$ is irrational.