

The Edges and Vertices of a Tree

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Tools

Definition 1. A graph G is an ordered pair $G = (V, E)$ where V is a set whose elements are called vertices and E is a set of 2-element subsets of V . The elements of E are called edges.

Definition 2. The degree of a vertex $v \in V$ is the number of edges containing that vertex.

Definition 3. Let G be a graph. A path in the graph G is a non-empty sequence of edges e_1, e_2, \dots, e_n , so that there are vertices v_1, v_2, \dots, v_{n+1} so that $e_i = \{v_i, v_{i+1}\}$.

Definition 4. A cycle is a path such that $e_i \neq e_{i+1}$ for any $i = 1, \dots, n - 1$, and such that $v_{n+1} = v_1$.

Definition 5. A graph is connected if, for any two vertices v and w , there is a path such that $v_1 = v$ and $v_n = w$.

Definition 6. A graph G is called a tree if it is connected, has at least one vertex, and has no cycles.

Task

Write a self-contained, clearly written proof of the following fact:

Theorem 1. Let T be a tree with n vertices. Then T has $n - 1$ edges.