Math 2001: Negation II (Katherine Stange, Spring 2018)

Task 1: Mark if each statement is true or false.
Task 2: Negate each statement.
Task 3 (sanity check): Mark if the negation is true or false.

1. There exists a negative integer which is a perfect square (F).
   
   Some possible negations:
   
   • There does not exist a negative integer which is a perfect square (T).
   • No negative integer is a perfect square (T).
   • Every negative integer is not a perfect square (T).

2. There exists a real number \( x \) such that \( x = -x \) (T).
   
   Some possible negations:
   
   • There does not exist a real number \( x \) such that \( x = -x \) (F).
   • No real number \( x \) satisfies \( x = -x \) (F).
   • All real numbers \( x \) satisfy \( x \neq -x \) (F).

3. Every integer is a rational number (T).
   
   Some possible negations:
   
   • There is an integer which is not a rational number (F).
   • There is an integer which is irrational (F).

4. All real numbers are either positive or negative (F).
   
   Some possible negations:
   
   • Not all real numbers are either positive or negative (T).
   • There are real numbers which are neither positive nor negative (T).
   • There are real numbers which are non-positive and non-negative (T).

5. If \( x \in \mathbb{Z} \) is even, then \( x > 3 \) (F).
   
   Some possible negations:
   
   • There is some \( x \in \mathbb{Z} \) which is even and satisfies \( x \leq 3 \) (T).
   • There is some \( x \in \mathbb{Z} \) which is even but satisfies \( x \leq 3 \) (T).

6. If \( x \in \mathbb{Z} \) is even, then \( x^2 \) is even (T).
   
   Some possible negations:
   
   • There is some \( x \in \mathbb{Z} \) which is even but \( x^2 \) is odd (F).
   • There is some \( x \in \mathbb{Z} \) which is even and \( x^2 \) is odd (F).

7. If \( x \in \mathbb{Z} \) is a negative perfect square, then \( x \) is prime (T).
   
   Some possible negations:
   
   • There is some \( x \in \mathbb{Z} \) which is a negative perfect square and not prime (F).
   • There is some \( x \in \mathbb{Z} \) which is a negative perfect square but not prime (F).
8. All integers are rational and positive (F).

Some possible negations:
• There is some integer which is irrational or non-positive (T).
• There is some integer which is either irrational or non-positive (T).
• There exists an integer which is either irrational or non-positive (T).

9. All integers are rational and real (T).

Some possible negations:
• There exists an integer which is either irrational or non-real (F).
• There is an integer which is either irrational or non-real (F).

10. Every integer is either prime or even (F).

Some possible negations:
• There is an integer which is neither prime nor even (T).
• There is an integer which is not prime and not even (T).

11. Every integer greater than 5 is either odd or composite (T).

Some possible negations:
• There is some integer greater than 5 which is neither odd nor composite (F).
• There is some integer greater than 5 which is even and not composite (F).