

Math 2001 Modular Arithmetic Exploration

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The later pages of this document show the full addition/multiplication tables for some small moduli.

Modular Arithmetic Functions: Addition

Mod 4

$$f : \mathbb{Z}/4\mathbb{Z} \rightarrow \mathbb{Z}/4\mathbb{Z}, f(x) = x + a$$

	0	1	2	3	function	injective?	surjective?	bijective?	diagram
0	0	1	2	3	$f(x)=x$				
1	1	2	3	0	$f(x)=x+1$				
2	2	3	0	1	$f(x)=x+2$				
3	3	0	1	2	$f(x)=x+3$				

Take a look at the other addition tables on the later pages. When is $f(x) = x + a$ injective/surjective/bijective? What does the diagram look like for these functions?

Modular Arithmetic Functions: Multiplication

Mod 4

$$f : \mathbb{Z}/4\mathbb{Z} \rightarrow \mathbb{Z}/4\mathbb{Z}, f(x) = ax$$

	0	1	2	3	function	injective?	surjective?	bijjective?	diagram
0	0	0	0	0	$f(x)=0x$				
1	0	1	2	3	$f(x)=1x$				
2	0	2	0	2	$f(x)=2x$				
3	0	3	2	1	$f(x)=3x$				

Mod 5

$$f : \mathbb{Z}/5\mathbb{Z} \rightarrow \mathbb{Z}/5\mathbb{Z}, f(x) = ax$$

	0	1	2	3	4	function	injective?	surjective?	bijjective?	diagram
0	0	0	0	0	0	$f(x)=0x$				
1	0	1	2	3	4	$f(x)=1x$				
2	0	2	4	1	3	$f(x)=2x$				
3	0	3	1	4	2	$f(x)=3x$				
4	0	4	3	2	1	$f(x)=4x$				

Mod 6

$$f : \mathbb{Z}/6\mathbb{Z} \rightarrow \mathbb{Z}/6\mathbb{Z}, f(x) = ax$$

	0	1	2	3	4	5	function	injective?	surjective?	bijjective?	diagram
0	0	0	0	0	0	0	$f(x)=0x$				
1	0	1	2	3	4	5	$f(x)=1x$				
2	0	2	4	0	2	4	$f(x)=2x$				
3	0	3	0	3	0	3	$f(x)=3x$				
4	0	4	2	0	4	2	$f(x)=4x$				
5	0	5	4	3	2	1	$f(x)=5x$				

Take a look at the other multiplication tables on the later pages. When is $f(x) = ax$ injective/surjective/bijjective? What does the diagram look like for these functions? There's a wide variety of behaviour! Make conjectures!

1 Addition Tables

Mod 4

	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Mod 5

	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

Mod 6

	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

Mod 7

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

Mod 8

	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5	6	7	0	1	2
4	4	5	6	7	0	1	2	3
5	5	6	7	0	1	2	3	4
6	6	7	0	1	2	3	4	5
7	7	0	1	2	3	4	5	6

Mod 9

	0	1	2	3	4	5	6	7	8
0	0	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8	0
2	2	3	4	5	6	7	8	0	1
3	3	4	5	6	7	8	0	1	2
4	4	5	6	7	8	0	1	2	3
5	5	6	7	8	0	1	2	3	4
6	6	7	8	0	1	2	3	4	5
7	7	8	0	1	2	3	4	5	6
8	8	0	1	2	3	4	5	6	7

Mod 10

	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	0
2	2	3	4	5	6	7	8	9	0	1
3	3	4	5	6	7	8	9	0	1	2
4	4	5	6	7	8	9	0	1	2	3
5	5	6	7	8	9	0	1	2	3	4
6	6	7	8	9	0	1	2	3	4	5
7	7	8	9	0	1	2	3	4	5	6
8	8	9	0	1	2	3	4	5	6	7
9	9	0	1	2	3	4	5	6	7	8

2 Multiplication Tables

Mod 4

	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

Mod 5

	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Mod 6

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

Mod 7

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

Mod 8

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	0	2	4	6
3	0	3	6	1	4	7	2	5
4	0	4	0	4	0	4	0	4
5	0	5	2	7	4	1	6	3
6	0	6	4	2	0	6	4	2
7	0	7	6	5	4	3	2	1

Mod 9

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8
2	0	2	4	6	8	1	3	5	7
3	0	3	6	0	3	6	0	3	6
4	0	4	8	3	7	2	6	1	5
5	0	5	1	6	2	7	3	8	4
6	0	6	3	0	6	3	0	6	3
7	0	7	5	3	1	8	6	4	2
8	0	8	7	6	5	4	3	2	1

Mod 10

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	0	2	4	6	8
3	0	3	6	9	2	5	8	1	4	7
4	0	4	8	2	6	0	4	8	2	6
5	0	5	0	5	0	5	0	5	0	5
6	0	6	2	8	4	0	6	2	8	4
7	0	7	4	1	8	5	2	9	6	3
8	0	8	6	4	2	0	8	6	4	2
9	0	9	8	7	6	5	4	3	2	1