List of Basic Logical Laws

These are listed on page 52 of Hammack 3rd edition, except the last two, which I find useful but aren’t there.

- Contrapositive Law: \((P \implies Q) = ((\neg Q) \implies (\neg P))\)
- DeMorgan’s Law I: \(\neg (P \land Q) = (\neg P) \lor (\neg Q)\)
- DeMorgan’s Law II: \(\neg (P \lor Q) = (\neg P) \land (\neg Q)\)
- Commutative Law for And: \(P \land Q = Q \land P\)
- Commutative Law for Or: \(P \lor Q = Q \lor P\)
- Distributive Law And over Or: \(P \land (Q \lor R) = (P \land Q) \lor (P \land R)\)
- Distributive Law Or over And: \(P \lor (Q \land R) = (P \lor Q) \land (P \lor R)\)
- Associative Law for And: \(P \land (Q \land R) = (P \land Q) \land R\)
- Associative Law for Or: \(P \lor (Q \lor R) = (P \lor Q) \lor R\)
- Simplification of Implies: \(P \implies Q = (\neg P) \lor Q\)
- Negation of Implies: \(\neg (P \implies Q) = P \land (\neg Q)\)

Using the laws above, manipulate the first expression to become the second one. State the laws you use as you use them.

1. \(\neg ((P \land Q) \lor R)\) is logically equivalent to \(((\neg P) \lor (\neg Q)) \land (\neg R)\)
   Hint: Apply DeMorgan’s twice.

2. \((P \lor Q) \implies R\) is logically equivalent to \(R \lor ((\neg P) \land (\neg Q))\)
   Hint: Simplify implies, then apply DeMorgan’s and then Commutativity.

3. \(\neg ((P \implies Q) \land R)\) is logically equivalent to \((P \land (\neg Q)) \lor (\neg R)\).
   Hint: Apply DeMorgan’s, then Negation of Implies.