Worksheet on Inverse Functions

November 17, 2015

1 Warmup

Let $f : A \to B$.

1. Write out the definition of *injectivity*.

2. Write out the definition of *surjectivity*.

3. Suppose $|A| > |B|$. Is it possible for f to be
   (a) injective?
   (b) surjective?
   (c) bijective?

4. Suppose $|A| < |B|$. Is it possible for f to be
   (a) injective?
   (b) surjective?
   (c) bijective?

5. Suppose $|A| = |B|$. Is it possible for f to be
   (a) injective?
   (b) surjective?
   (c) bijective?

2 The identity function

Definition 1. *Let A be a set. We write 1_A for the* identity function on A*, given by $1_A(a) = a$ for all $a \in A$.*

1. Let $A = \{1, 2, 3\}$. Draw the arrow diagram of 1_A.

2. In general, is 1_A injective, surjective or bijective?

3 Inverse functions

Definition 2. *Let $f : A \to B$ be a function. Let $g : B \to A$ be a function. Then we say that g is the* inverse of f*, and denote it f^{-1}, if $g \circ f = 1_A$ and $f \circ g = 1_B$.*

1. For each function f, draw the arrow diagram of f. Draw the arrow diagram of f^{-1} or explain why f^{-1} doesn't exist.

   (a) $f : \{A, B, C\} \to \{1, 2, 3\}$ given by $f(A) = 1, f(B) = 3, f(A) = 2$.

   (b) $f : \{A, B, C\} \to \{1, 2\}$ given by $f(A) = 1, f(B) = 1, f(A) = 2$.

   (c) $f : \{A, B\} \to \{1, 2, 3\}$ given by $f(A) = 1, f(B) = 2$.

2. Does $f : \mathbb{Z} \to \mathbb{Z}$ given by $f(x) = x + 1$ have an inverse? What is it?

Theorem 1. *Let $f : A \to B$ be bijective. Then f has an inverse.*