Worksheet on Relations and Equivalence Relations and Classes

November 6, 2015

1 Review of Reflexivity, Symmetry and Transitivity

For each of the following, indicate if it is reflexive (R), symmetric (S) and/or transitive (T). Mark the equivalence relations with an E.

1. The relation $=$ on the real numbers.
2. The relation $<$ on the rational numbers.
3. The relation $\subseteq$ on the power set of a set.
4. The relation *'has the same cardinality'* on the power set of a set.
5. The relation *'has smaller cardinality'* on the power set of a set.
6. The relation $\neq$ on the integers.

2 Equivalence classes

Definition 1. *Let A be a set, and let R be an equivalence relation on A. For any $a \in A$, the* equivalence class containing *a (which we denote by $[a]$), is the set of all elements in A that are related to a. In symbols,*

$$[a] = \{b \in A : (a, b) \in R\}.$$

For each equivalence relation, list the equivalence classes.

1. The relation $=$ on the integers.

2. The relation *has the same size* on the power set of $\{a, b\}$.

3. The relation on $A = \{1, 2, 3, 4\}$ given by

$$R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1)\}.$$

4. The relation on $\mathbb{Z}$ defined by

$$R = \{(a, b) : a, b \in \mathbb{Z}, a - b \text{ is even}\}.$$

5. The relation on $\mathbb{Z}$ defined by

$$R = \{(a, b) : a, b \in \mathbb{Z}, a - b = 5c \text{ for some } c \in \mathbb{Z}\ \}.$$

3 Some properties

Theorem 1. *Suppose R is an equivalence relation on a set A. Suppose also that $a, b \in A$. Then $[a] = [b]$ if and only if $(a, b) \in R$.*

Proof. □