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Let this largest integer be n . **Then $n + 1$ is an integer.**

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Suppose, for a contradiction, that there were a largest integer. Let this largest integer be n . Then $n + 1$ is an integer. **But $n + 1 > n$, contradicting the assumption that n was the largest integer.** □

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Suppose, for a contradiction, that there were only finitely many such primes. Call the largest such prime p_0 . Let p be a prime larger than p_0 . **Then $p + 2$ is prime.**

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Suppose, for a contradiction, that there were only finitely many such primes. Call the largest such prime p_0 . Let p be a prime larger than p_0 . Then $p + 2$ is prime. **Since $p + 2 > p_0$, then $p + 4$ is prime.**

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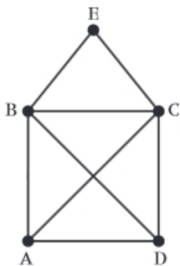
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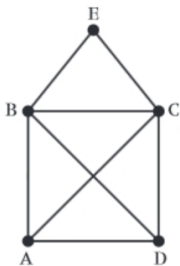
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The number $3p > p$ is one such an odd composite. □



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Consider the picture at left. It is impossible to traverse this diagram along the edges in a cycle (ending where you begin) using each edge exactly once.

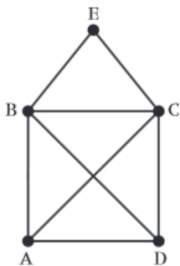


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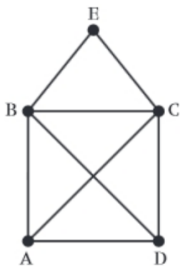


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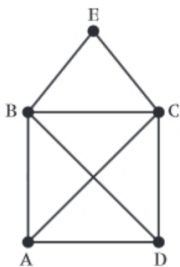


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For a contradiction, suppose there were such a cycle. In such a cycle, the cycle would leave each vertex as many times as it entered it. **Since each edge is used exactly once, the number of edges touching a vertex must therefore be even.**



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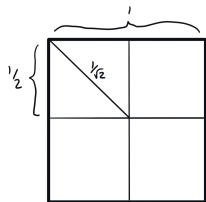
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Consider a unit square with sides of length 1. Suppose 5 points are placed inside this square. Then there exist two points x and y among these five, such that the distance between x and y is less than or equal to $1/\sqrt{2}$.

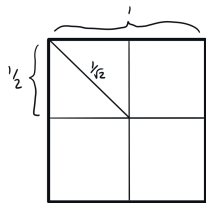


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Divide the square into four quadrants as shown.

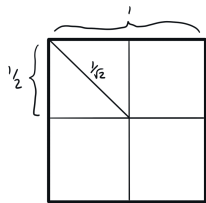


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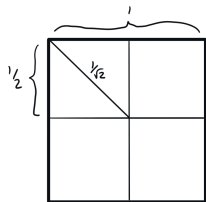


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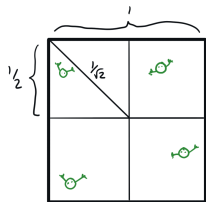


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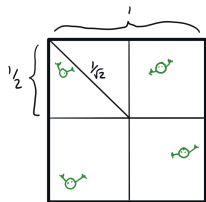


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