Worksheet on Functions and Composition

March 18, 2020

1. For the following functions, determine which compositions are defined, and for each composition that is defined, determine the composition.

   $f : \{1, 2, 3\} \to \{a, b\}$

x	$f(x)$
1	a
2	b
3	b

   $g : \{a, b\} \to \{1, 2\}$

x	$g(x)$
a	1
b	2

   $h : \{a, b, c\} \to \{1, 2, 3\}$

x	$h(x)$
a	1
b	1
c	3

2. For each of the following functions, determine if it has an inverse and what the inverse should be. If it has no inverse, explain why. If you give an inverse, prove it (meaning, compute the composition in both orders and check it is the identity).

   (a) $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^3 + 1$.

   (b) $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2 + 1$.

   (c) $g : \{a, b, c\} \to \{1, 2, 3\}$ given by

x	$g(x)$
a	1
b	1
c	3

   (d) $g : \{a, b, c\} \to \{1, 2, 3\}$ given by

x	$g(x)$
a	1
b	3
c	2