Definition 1. *A set A is a* subset *of a set B, denoted $A \subset B$, if every element of A is an element of B.*

0.1 Subsets

1. List all the possible subsets of $\{a, b\}$. (Hint: there are four.)

0.2 Cartesian products

Have Section 1.2 of Hammack handy (on your smartphone or whatever).

1. Write the cartesian product $A \times B$ where $A = \{1, 2\}$ and $B = \{a, b\}$.

2. Explain why, for the example above, $A \times B \neq B \times A$.

3. Give an example of two sets A and B where $A \times B = B \times A$.

4. Determine $A \times \emptyset$, where $A = \{1, 2\}$.

5. Give an example of two sets A and B where $A \times B = B \times A$ but $B \neq A$.

6. If $|A| = n$ and $|B| = m$, then what is $|A \times B|$?

7. Give set builder notation for $\mathbb{R} \times \mathbb{R}$.

8. The graph of a function is a subset of $\mathbb{R} \times \mathbb{R}$. That is,

$$\{(x, f(x)) : x \in \mathbb{R}\} \subset \mathbb{R} \times \mathbb{R}.$$

   Of the examples below, draw the graph. Which of these graphs is **itself** a Cartesian product? If it is a Cartesian product, find A and B so that the graph is $A \times B$.

   (a) $f(x) = x^2$

   (b) $f(x) = x$

   (c) $f(x) = 3$

0.3 Getting crazy

1. Let S be the set of sets that do not contain themselves. In notation,

$$S = \{X : X \notin X\}.$$

   (a) Give an example of an element of S.

   (b) Give an example of something not in S.

   (c) Is $S \in S$?

2. We say that a set X satisfies the *Well-Ordering Principle* if every non-empty subset of X has a least element. (This only makes sense in contexts where "least" is defined, such as numbers.) Which of the following sets satisfy the Well-Ordering Principle? If it fails, give an example that demonstrates this.

   (a) $\mathbb{Z}$

   (b) $\mathbb{N}$

   (c) $\mathbb{R}$

3. The Well-Ordering Principle can be used to prove that there are no positive integers strictly between 0 and 1. Can you prove it? Here's an outline:

   Let S be the set of positive integers strictly between 0 and 1. Suppose S is non-empty.

   Therefore we have reached a contradiction (something impossible). Therefore our supposition that S is non-empty must have been false.

   Therefore S is empty.