Definition 1. *A set A is a* subset *of a set B, denoted $A \subset B$, if every element of A is an element of B.*

Definition 2. *Let A and B be sets. The Cartesian product $A \times B$ is defined as follows:*

$$A \times B := \{(a, b) : a \in A, b \in B\}.$$

0.1 Subsets

1. List all the possible subsets of $\{a, b\}$. (Hint: there are four.)

0.2 Cartesian products

1. Write the cartesian product $A \times B$ where $A = \{1, 2\}$ and $B = \{a, b\}$.

2. Explain why, for the example above, $A \times B \neq B \times A$.

3. Give an example of two sets A and B where $A \times B = B \times A$.

4. Determine $A \times \emptyset$, where $A = \{1, 2\}$.

5. Give an example of two sets A and B where $A \times B = B \times A$ but $B \neq A$.

6. If $|A| = 5$ and $|B| = 7$, then what is $|A \times B|$?

7. Give set builder notation for $\mathbb{R} \times \mathbb{R}$.

8. The graph of a function is a subset of $\mathbb{R} \times \mathbb{R}$. That is,

   $$\{(x, f(x)) : x \in \mathbb{R}\} \subset \mathbb{R} \times \mathbb{R}.$$

   Of the examples below, draw the graph. Which of these graphs is **itself** a Cartesian product? If it is a Cartesian product, find A and B so that the graph is $A \times B$.

   (a) $f(x) = x^2$

   (b) $f(x) = x$

   (c) $f(x) = 3$

0.3 Getting crazy

1. Let S be the set of sets that do not contain themselves. In notation,

$$S = \{X : X \notin X\}.$$

   (a) Can you give an example of an element of S?

   (b) Can you give an example of something not in S?

   (c) Is $S \in S$?

2. We say that a set X satisfies the *Well-Ordering Principle* if every non-empty subset of X has a least element. (This only makes sense in contexts where "least" is defined, such as numbers.) Which of the following sets satisfy the Well-Ordering Principle? If it fails, give an example that demonstrates this.

   (a) $\mathbb{Z}$

   (b) $\mathbb{N}$

   (c) $\mathbb{R}$

3. Challenge: Write down, with finite notation (no "..."), e.g. setbuilder notation or symbols we have used so far,

   (a) the biggest *finite* set you can think of.

   (b) the "biggest" infinite set you can think of.

   Compare to your friends. Whose is bigger? How can you tell? If your friend beat you, make yours bigger!