MATHEMATICS 2001
INDIVIDUAL WORK DUE FRIDAY FRIDAY APR 6 AT 8 AM

General information. You can work with others as long as you write up your solutions yourself (not copying). Hand this in on canvas as with previous groupwork.

Please spend approximately 2 hours on this homework. If you don't finish all the proofs, hand in what you have.

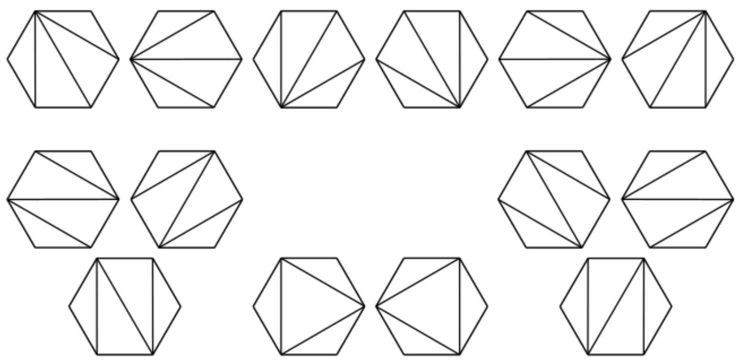
Triangulations

Let P be a convex polygon with n sides. Convex, for us, means the interior angles are all less than 180 degrees. In such a polygon, it is possible to draw a diagonal between any two non-consecutive vertices, which passes through the interior.

Definition 1. *A* triangulation *of P is a collection of such diagonals which do not cross one another, and divide the interior into triangular regions. If such a triangulation exists, we say that it* triangulates *P.*

Definition 2. *Given a convex polygon P with n sides, and a triangulation of that polygon, a triangle of the triangulation is called an* ear *if it shares two sides with the original polygon.*

Here are a whole bunch of ways to triangulate a hexagon. Most have two ears, but the ones in the lower middle have three ears.



Definition 3. *The* dual graph *of a triangulation is the graph which has one vertex for each triangle, and two vertices are joined exactly when the two triangles share a side.*

Prove the following theorems. You may wish to re-order them, as some may depend upon others.

Theorem 1. *The dual graph to any triangulation is a tree whose vertices have degree at most 3.*

Theorem 2. *Any triangulation of a convex polygon with n sides has at least two ears.*

Theorem 3. *Any convex polygon with n sides can always be triangulated in at least one way.*

Theorem 4. *Any triangulation of a convex polygon with n sides breaks it into exactly $n-2$ triangles.*

Theorem 5. *Any triangulation of a convex polygon with n sides uses exactly $n-3$ diagonals.*

Here's something to ponder, if you like: how many different triangulations of an n-gon are possible? The hexagon had 14, as in the picture above.