

MATHEMATICS 2001
INDIVIDUAL WORK DUE FRIDAY APR 20 AT 8 AM

General information. You can work with others as long as you write up your solutions yourself (not copying). Hand this in on canvas as with previous groupwork.

Please spend approximately 2 hours on this homework. If you don't finish all the proofs, hand in what you have.

ASSIGNMENT

Prove the following theorems.

Theorem 1. *Suppose A , B , and C are sets. Suppose that $A \subseteq B$ and $B \subseteq C$. Then $A \subseteq C$.*

Hint: Turn the final $A \subseteq C$ into an *if-then* statement, using the definition of \subseteq (the definition is in Hammack if you've forgotten).

Theorem 2. *Let X be a set. The relation \subseteq on subsets of $\mathcal{P}(X)$ is reflexive and transitive, but not symmetric.*

Hint: There are three tasks. The hardest one is a consequence of Theorem 1 above.

Theorem 3. *Let A , B and C be sets. Then*

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

Hint: An equality of sets is actually two inequalities:

$$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C),$$

and

$$A \cap (B \cup C) \supseteq (A \cap B) \cup (A \cap C).$$

So break the proof into these two tasks.

Final task: Can you explain the relationship of the above theorem with the logical equivalence:

$$P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)?$$