

MATHEMATICS 2001
GROUPWORK DUE SEPTEMBER 9

TASKS

Reminder: you should produce a Groupwork Report (handwritten is fine) and a PDF uploaded to D2L (typset, LaTeX or Word or whatever).

Reminder: elect a leader, scribe and presenter.

- (1) **Main Task 1: Take up homework done so far.** Note: done right, this will take a long time, perhaps an hour. Open up the website and go through all the days since you last met, and pull out everyone's homework. For **each** assigned homework task, **each** person shares their answers with the group. Ask each other questions until everyone has understood everyone else's answers and any disagreement have been resolved.
- (2) **Main Task 2: Group Homework.**

You should look up the definition of divides, Definition 4.4, and study it (this means, in particular, do some examples to demonstrate which things do and don't divide each other).

Also, here's a new definition to work with:

Definition 1. *Let n be an integer. Two integers a and b are said to be congruent modulo n if n divides $a - b$.*

Study this definition by generating your own examples. For example, 1 and -3 are congruent modulo 4 but not congruent modulo 5.

Work together to write your best proofs of the following theorems:

- (a) Suppose a and b are both odd. Then they are congruent modulo 2.
- (b) Suppose a , b and c are integers, and suppose that $a \mid b$ and $a \mid c$. Then $a^2 \mid bc$.
- (c) Suppose a is an even integer, and c is a multiple of b . Then $2b \mid ac$.
- (d) Let $m, x, y \in \mathbb{Z}$. Suppose x and y are congruent modulo m . Then x^2 and y^2 are congruent modulo m .
- (e) The sum of an even number of odd numbers is even. (Hint: example: $3+5+7+1 = 16$ is even since we added 4 (an even number of) odd numbers. You may need to use notational techniques like summation notation and indices (which you should have encountered in calculus) to write a proof. If you are having trouble, write a proof that the sum of 4 odd numbers is even, then write a proof that the sum of 6 odd numbers is even. Then try the general case again. If you can't do the general case, please hand in the 4 and 6 cases you just did.)

You must work together on these and **not** show up at the group meeting having done them ahead of time. Do **not** divide up the work. Instead, spend 1-2 minutes each thinking silently, and then begin to construct the proof on a single sheet of paper collaboratively, discussing as you go. You will need to change things as you go, so feel free to do multiple drafts. Do not move on to the next until you all feel satisfied with the one you are working on. If you find yourself ahead of the group, take on a socratic role, guiding your classmates through questions. If you find yourself getting lost in the group, tell your groupmates you are feeling lost and ask questions.

- (3) Fill out your groupwork report and have everyone sign. **This is due in class.**
- (4) The scribe will prepare a PDF of your proofs to hand in on D2L. **I appreciate getting these early on Friday so I can look through them.**