## MATHEMATICS 2001 GROUPWORK DUE SEPTEMBER 16

## TASKS

Reminder: you should produce a Groupwork Report (handwritten is fine) and a PDF uploaded to D2L (typset, LaTeX or Word or whatever).

Reminder: elect a leader, scribe and presenter.

- (1) Reminder: You should elect a scribe and presenter who has not yet scribed or presented. If you were elected presenter but did not actually present in class, that doesn't count as having presented, so you may be elected again.
- (2) Main Task 1: Take up homework done so far. Note: done right, this will take a long time, perhaps an hour. Open up the website and go through all the days since you last met, and pull out everyone's homework. For each assigned homework task, each person shares their answers with the group. Ask each other questions until everyone has understood everyone else's answers and any disagreement have been resolved.

## (3) Main Task 2: Group Homework.

- (a) Can you find a set which is a subset of itself, but not an element of itself?
- (b) Can you find a set which is an element of itself, but not a subset of itself?
- (c) Can you think of an object x such that  $x \in \{x\}$ ?
- (d) Consider the following expressions in terms of sets A, B and C. Which of them are necessarily always equal?
  - (i)  $A \cup (B \cap C)$
  - (ii)  $A \cap (B \cup C)$
  - (iii)  $(A \cap B) \cup (A \cap C)$
  - (iv)  $(A \cup B) \cap (A \cup C)$

Why or why not? If you say two of them are not necessarily equal, then give examples (concrete A, B and C) that demonstrate that it can fail. If they are equal, you can give a justification of any reasonably convincing sort (e.g. Venn diagram with accompanying explanation). Note: with four items, there are six pairs to compare. Note: I would be ok with you splitting up the work on this and taking turns explaining to each other.

- (e) Write the following statement symbolically in terms of set builder notation and the subset symbol: The set of integers divisible by 4 is a subset of the set of integers divisible by 2.
- (f) Prove the statement from the last item. Hint: rewrite the statement in *if-then* form, then use direct proof.
- (g) Suppose A and B and C are sets. Suppose that  $A \subseteq B$  and  $B \subseteq C$ . Prove that  $A \subseteq C$ . Hint: as in the last question, turn  $A \subseteq C$  into an *if-then* statement.
- (h) Consider the statement: If x is the square of an even integer, then x is divisible by four. Write this statement as an inclusion of sets, i.e. use set-builder notation and the subset symbol.
- (4) Fill out your groupwork report and have everyone sign. This is due in class.
- (5) The scribe will prepare a PDF of your proofs to hand in on D2L. I appreciate getting these early on Friday so I can look through them.