# MATHEMATICS 2001
# GROUPWORK DUE FRIDAY FRIDAY MAR 9 AT 8 AM

**General information.** You should produce two things to hand in: a groupwork report using the website template (handwritten; hand in in class), and a PDF of four proofs uploaded to canvas, which must be **typset** in some way (not handwritten).

Choose someone to fill each of these roles: leader, scribe, presenter. The leader's job is to keep everyone on task. The scribe's job is to create the two documents to hand in. The presenter's job is to present solutions in class (if time permits; not every group will present). Each week, these tasks should rotate, so everyone gets a turn.

Please spend approximately 2 hours on this homework. Work collaboratively, as you did last week. If you don't finish all the proofs, hand in what you have.

**Task 1: Proof by Contrapositive.** *Proof by Contrapositive* is a type of proof you can apply to an if-then statement. The method is this: rephrase the theorem as its contrapositive, then prove that. This is well-explained in Hammack, Chapter 5. In particular, look at the example at the bottom of page 103. Prove the following by contrapositive:

**Theorem 1.** *Let $n$ be an integer. If $n^2$ is odd, then $n$ is odd.*

**Task 2: Fill in our graph theory lemma.** Recall that in order to prove that trees on $n$ vertices have $n-1$ edges, we used the following lemma. Please prove this lemma, using the formal definition of a tree.

**Theorem 2.** *If an edge is deleted from a tree, the resulting graph is no longer connected.*

**Task 3: Practice proof by induction.** Prove the following theorems by induction. If you have another way to prove them, you can hand in a second proof to compare! Note: as usual, spend a maximum of 2 hours. If you can't do all the proofs, choose your favourites.

**Theorem 3.** *The number of binary strings of length $n$ is $2^n$.*

**Theorem 4.** *For any integer $n$,*

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}.$$

Note, for clarity:

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{n(n+1)}.$$

**Definition 1.** *In a graph $G$, a vertex of degree 1 is called a* leaf.

**Theorem 5.** *Every tree with at least 2 vertices has at least 2 leaves.*

**Theorem 6.** *Let $T$ be a tree. The vertices of $T$ can be coloured using two colours, so that no edge connects vertices of the same colour.*

**Theorem 7.** *Any set of $n$ elements has exactly $2^n$ subsets.*