

MATHEMATICS 2001
GROUPWORK DUE FRIDAY JANUARY 26 AT 8 AM

TASKS

You should produce two things to hand in:

1) a Groupwork Report. There is a sheet to fill out under Resources on the website; handwritten is fine. You can hand this in in class. It doesn't need to be insanely detailed, but there is such a thing as too brief. Tell me about your experience.

2) A PDF uploaded to canvas, with the solutions you have created (typset, LaTeX or Word or whatever).

- (1) **Electing a leader, scribe and presenter for this week.** Choose someone to fill each of these roles. The leader's job is to keep everyone on task. The scribe's job is to create the two documents to hand in. The presenter's job is to present solutions in class (if time permits; not every group will present). Each week, these tasks will rotate, so everyone gets a turn.
- (2) **Main Task 1: Study questions.** If anyone has any questions about the material of the course, discuss and help one another.
- (3) **Main Task 2: Group Homework.** Please spend approximately 1 hour on this homework. This should hopefully give you time to tackle the three main proofs. If those took less than an hour, try the challenge problem.

You must work together on these and **not** show up at the group meeting having done them ahead of time. Do **not** divide up the work. Instead, spend 1-2 minutes each thinking silently, and then begin to construct the proof on a single sheet of paper collaboratively, discussing as you go. You will need to change things as you go, so feel free to do multiple drafts. Do not move on to the next until you all feel satisfied with the one you are working on. If you find yourself ahead of the group, take on a socratic role, guiding your classmates through questions. If you find yourself getting lost in the group, tell your groupmates you are feeling lost and ask questions.

You should look up the definition of **odd**, Definition 4.2, and study it. Also look up **divides**, Definition 4.4, and study it (this means, in particular, do some examples to demonstrate which things do and don't divide each other).

Also, here's a new definition to work with:

Definition 1. *Let n be an integer. Two integers a and b are said to be congruent modulo n if n divides $a - b$.*

Study this definition by generating your own examples. For example, 1 and -3 are congruent modulo 4 but not congruent modulo 5.

You may find Chapter 4.3 useful as a resource for what follows.

Work together to write your best proofs of the following three theorems:

- (a) Suppose that n is an odd integer. Then n^3 is an odd integer.
 - (b) Suppose a , b and c are integers, and suppose that $a \mid b$ and $a \mid c$. Then $a^2 \mid bc$.
 - (c) Suppose a and b are both odd. Then they are congruent modulo 2.
- (4) Here's an additional challenge problem, to give you a sense of where you could go in future. Prove this theorem: *The sum of an even number of odd numbers is even.* (Hint: example: $3 + 5 + 7 + 1 = 16$ is even since we added 4 (an even number of) odd numbers. You may

need to use notational techniques like summation notation and indices (which you should have encountered in calculus) to write a proof. If you are having trouble, write a proof that the sum of 4 odd numbers is even, then write a proof that the sum of 6 odd numbers is even. Then try the general case again. If you can't do the general case, please hand in the 4 and 6 cases you just did.)

- (5) Fill out your groupwork report and have everyone sign. **This is due in class.**
- (6) The scribe will prepare a PDF of your proofs to hand in on canvas. **Hand in on canvas by 8 am Friday.** There's a link to canvas from the Resources page of the website, and I will set up a homework collection box there.