MATHEMATICS 2001
GROUPWORK DUE FRIDAY FRIDAY FEB 2 AT 8 AM

General information. You should produce two things to hand in: a groupwork report (handwritten; hand in in class), and a PDF of four proofs uploaded to canvas, which must be **typset** in some way (not handwritten).

Choose someone to fill each of these roles: leader, scribe, presenter. The leader's job is to keep everyone on task. The scribe's job is to create the two documents to hand in. The presenter's job is to present solutions in class (if time permits; not every group will present). Each week, these tasks should rotate, so everyone gets a turn.

Please spend approximately 2 hours on this homework. In what follows, you will produce four proofs total. Work collaboratively, as you did last week. Note: do not use any outside resources besides the video that I point you to below.

Important definitions. Here are some useful definitions:
(1) A real number x is said to be *rational* if it can be written in the form $x = a/b$ where a and b are integers.
(2) A real number x is said to be *irrational* if it is not rational.

First, play with these definitions. For example, discuss why integers are rational. It is also a known fact that π and e are irrational (although the proofs are complicated).

First two proofs. Work together to write your best proofs of **two** of the following theorems (**you may pick which two**):
(1) The negative of a rational number is rational.
(2) The sum of two rational numbers is rational.
(3) The difference of two rational numbers is rational.
(4) The product of two rational numbers is rational.
(5) The quotient of two rational numbers is rational (assuming the denominator is non-zero).

Third proof. The next three theorems are actually consequences of the previous theorems.
(1) The sum of a rational number and an irrational number is irrational.
(2) The product of a rational number and an irrational number is irrational.
(3) The quotient of a rational number and an irrational number is irrational (assuming a non-zero denominator).

First, discuss all together why these theorems follow from the first group of theorems. Your task is to work together to write down this reasoning as a formal proof of **one** of these facts. **You may choose which one.** (Note: If you feel stuck, move on to the last task and come back to this one later.)

Fourth proof. Now, we will show in class sometime this week that $\sqrt{2}$ is irrational. You may assume that $\sqrt{2}$ is irrational as a known fact, for what follows. You may also assume all the theorems stated above. Here's your theorem:

Theorem 1. *In between any two rational numbers, there is an irrational number.*

For this proof, watch the Khan Academy video (linked on website next to groupwork assignment link), which explains a proof in a casual way. Pause and replay as often as needed. Write down a formal version of the argument, i.e. give a formal proof. Warning! Sal is pretty casual and forgets to point out or check various important things. It's your job to make it a good solid proof.