Introduction

The famous Four Colour Theorem answers the following question: given a map with numerous contries (each country is connected, i.e. just one piece), how many colours are needed, at minimum, to colour it so that no two bordering countries are the same colour?

It turns out the answer is that for all possible maps, four is the largest number of colours needed. Draw a few simple examples and four-colour them (you can use four labels if you don't have colours).

Some information about the map is actually irrelevant to the problem at hand: the exact squigglyness of the border, for example, or the size of the country. What matters is exactly: a list of countries, plus which countries touch, and which don't. Therefore it is natural to introduce the notion of a *graph*, which is a set of *vertices* (countries), each pair of which is either *connected* (bordering) or not. We draw an edge to indicate connectedness. (Think of this as a sort of schematic of which countries border which.) Draw a map and the corresponding graph:

Warm-up

1. The graph having n vertices connected in a cycle is called C_n. Can you get C_n from a map?
2. A graph having no cycles is called a tree. Can you get a tree from a map?
3. If a tree has n vertices, how many edges does it have? Why?

Exploration

1. Can you get any possible graph from a map? Or do maps produce just a special sub-class of graphs?
2. If a graph produced by a map has n vertices and m edges, how many regions does it break the paper up into?
3. Can you prove the 4-colour theorem? The 5-colour theorem? The 6-colour theorem?