Theorem. Every amount of postage $n \geq 5$ cents can be made by a combination of 2-cent and 5-cent stamps.

Pf. We will induct on n.

Base Case: 5 can be formed of 2- and 5- cent stamps since $5 = 5$.
6 " " " " $6 = 2 + 2 + 2$.

Inductive Step: Let $n > 6$.
Suppose all amounts $5 \leq m < n$ can be formed of 2 and 5 cent stamps.

Then $n - 2 = 2k + 5\ell$ for some $k, \ell \in \mathbb{N}$, since $5 \leq n-2 < n$.

So $n = 2(k+1) + 5\ell$.

So n can be formed of 2 and 5 cent stamps.

$\blacksquare$

Theorem. Every amount of postage $n \geq 5$ cents can be made by a combination of 2-cent and 5-cent stamps.

Rough work..

$5 = 5$ Base

$+2$

$6 = 2+2+2$ Base

$7 = 5+2$ Ind $+2$

$+2$

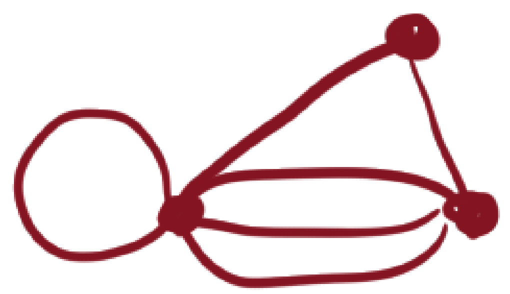
$8 = 2+2+2+2$ Ind

$+2$

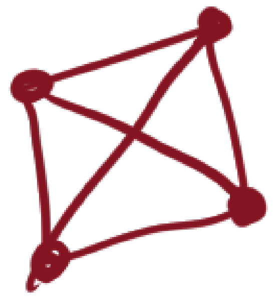
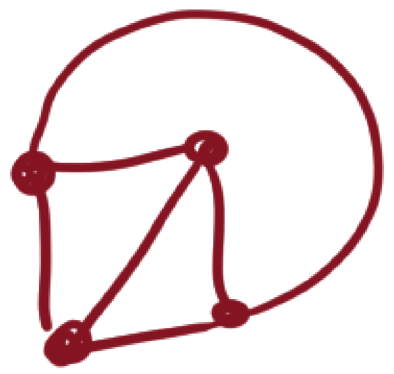
$9 = 5+2+2$ Ind

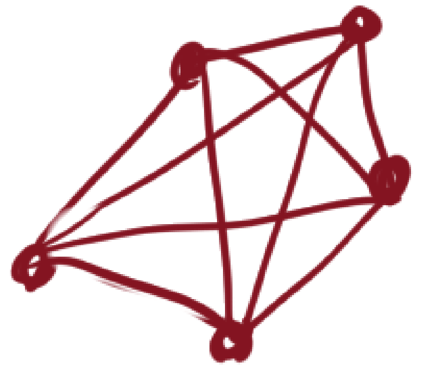
$10 = 5+5 = 2+2+2+2+2$ $-2-2$ $+5$

Def^n A *multigraph* allows *multiple edges* and *loops*. *Example*.

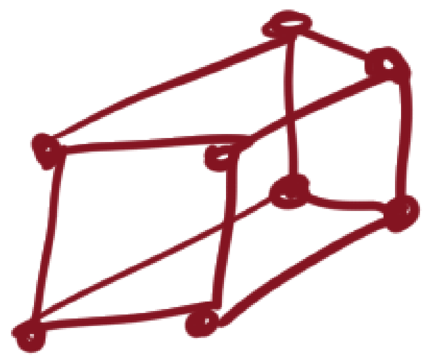
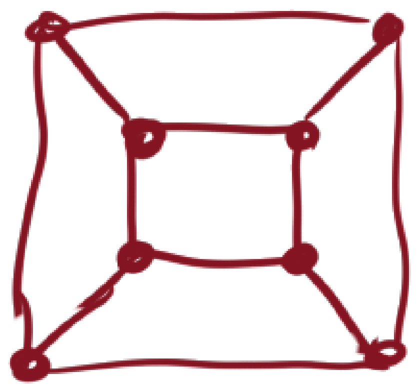
  

Def^n A (multi)graph is *planar* if it can be drawn in the plane without crossing edges.

Eg.  is planar because  is the same graph.

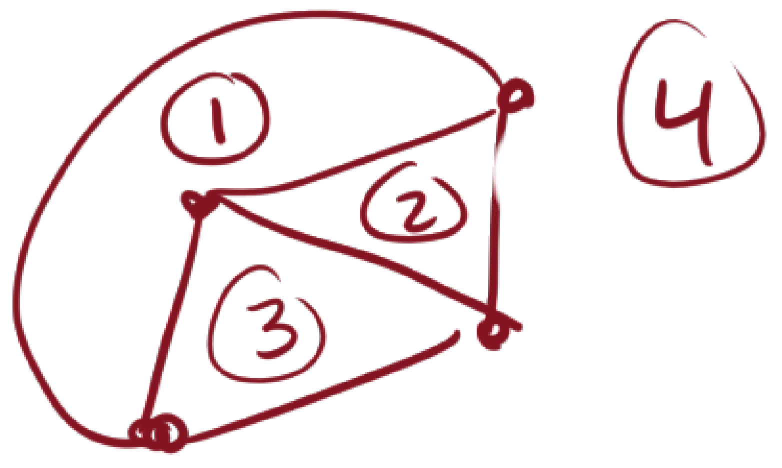
 is not.

Polyhedra $\Longrightarrow$ connected planar graphs

 $\Longrightarrow$ 

Def^{n} A *face* of a planar (multi) graph is a region circumscribed by a cycle of edges.
(It can include ∞.) = pieces left if you cut along the graph

Eg.

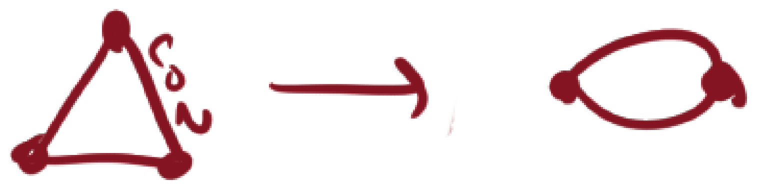


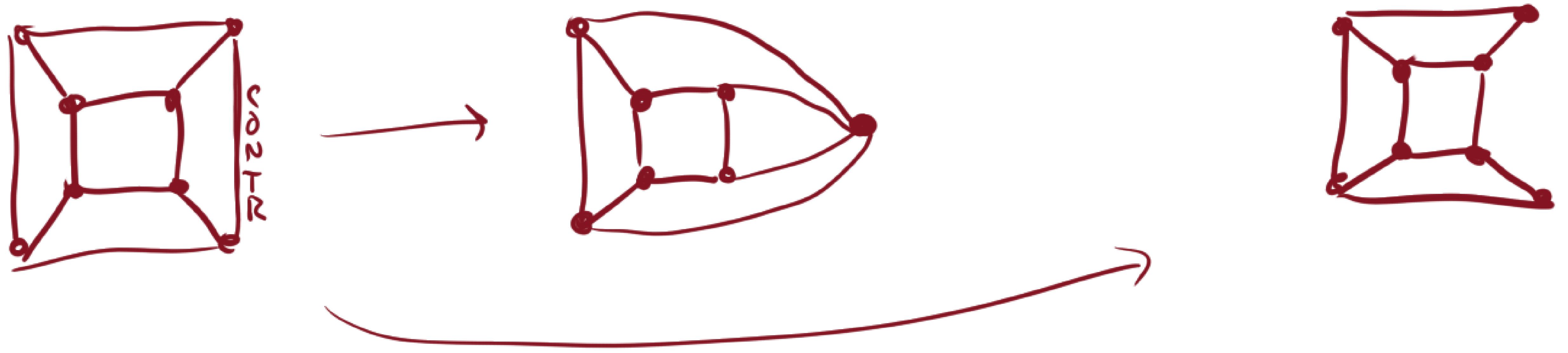
6 edges
4 faces
4 vertices

$$4-6+4=2. \checkmark$$

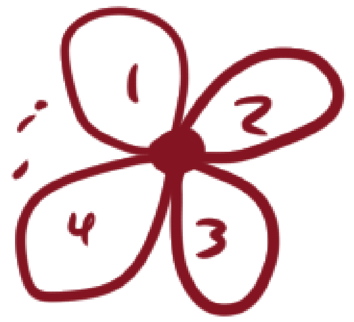
Theorem. Any connected planar multi graph with E edges, V vertices and F faces satisfies $\boxed{V-E+F=2.}$

Induct on # vertices!





Proof. We will induct on # of vertices.

Base Case: A planar connected multigraph on 1 vertex has n loops: 

1 vertex
n edges
n+1 faces

And

$$1 - n + (n+1) = 2$$

Inductive Step: Let $n > 1$. Suppose any connected planar multigraph with $n-1$ vertices satisfies $V - E + F = 2$.

Consider a conn. pl. multigraph w/ n ve[illegible].

Choose an edge connecting 2 vertices (this exists since $n > 1$ & graph connected).

Contract it.

The resulting multigraph has 1 fewer vertex and 1 fewer edge.

So $V - E + F = 2$ still holds!

$\boxtimes$