Relations

Informally, a property of an ordered pair of elements.

Example: $<$ is a relation on $\mathbb{R}$

"$a < b$" = a is less than b.

Example: $=$ on $\mathbb{Z}$

divides ("$|$") on $\mathbb{Z}$

Example: "Has the same cardinality" on all sets.

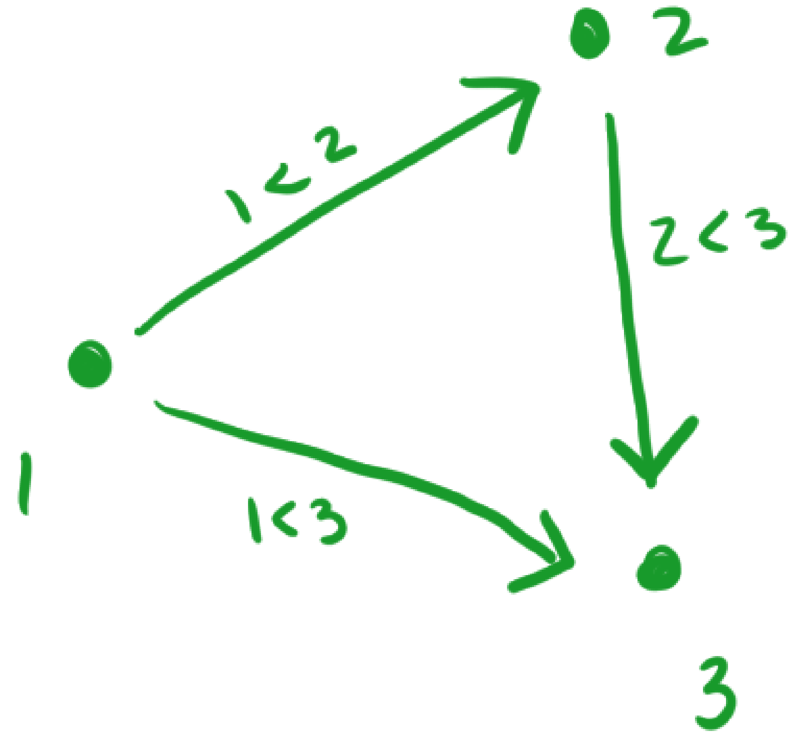
Def^n A relation on a set A is a subset R of $A \times A$.

Example: $<$ on $\{1,2,3\} = A$

is $\{(1,2),(1,3),(2,3)\} \subseteq A \times A$

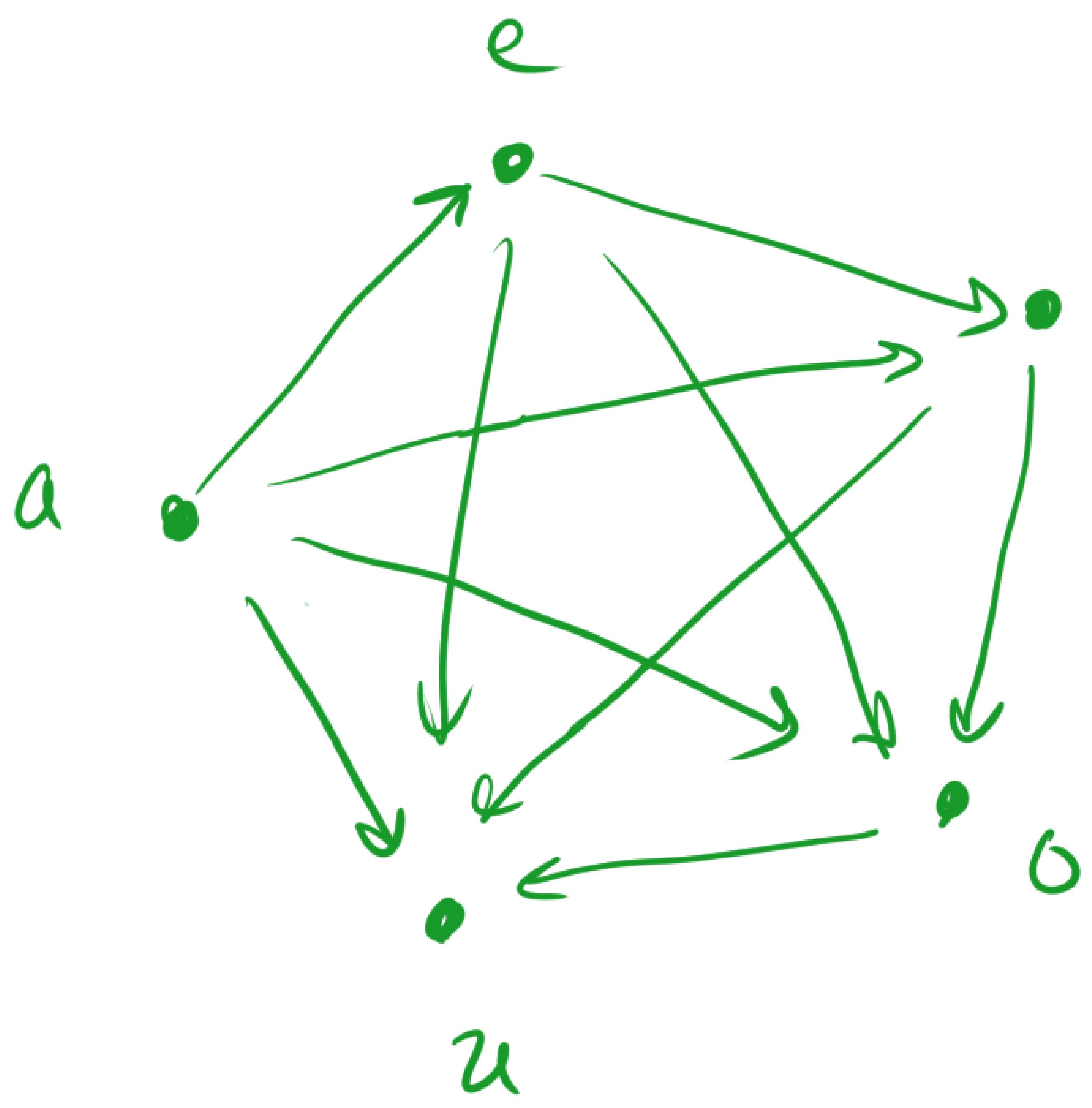
Ex. $<$ on $A = \{1, 2, 3\}$

is $\{(1,2), (2,3), (1,3)\}$



Examples. "comes before in alphabetical order" on the set of vowels $\{a, e, i, o, u\}$.

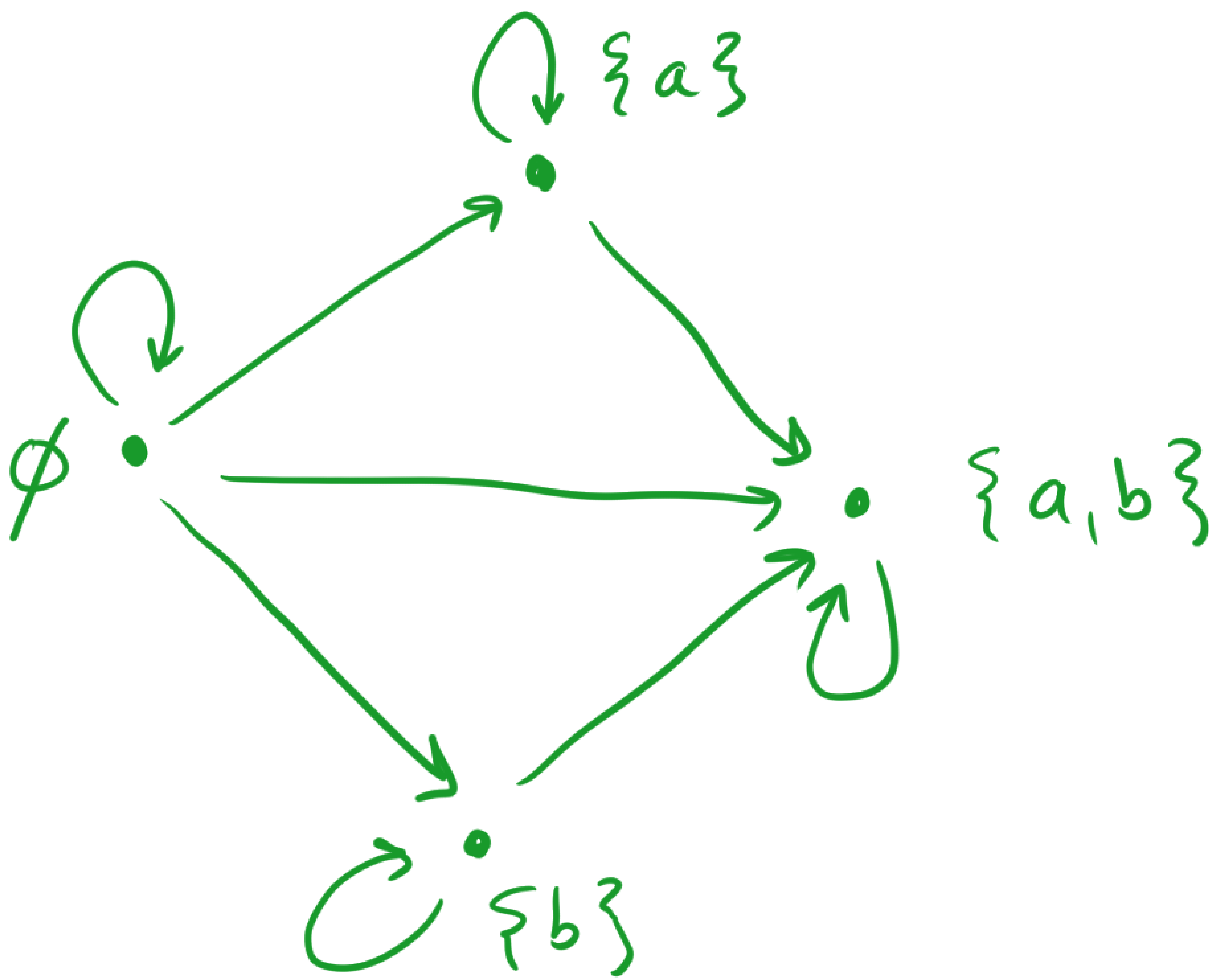
$$\{(a,e), (a,i), (a,o), (a,u), (e,i), (e,o), (e,u), (i,o), (i,u), (o,u)\}$$



Example. $S = \{a, b\}$

"$\subseteq$" on $\mathcal{P}(S)$

$\{(\emptyset, \{a\}), (\{a\}, \{a,b\}),$
$(\emptyset, \{b\}), (\{b\}, \{a,b\}),$
$(\emptyset, \{a,b\}), (\emptyset, \emptyset),$
$(\{a\}, \{a\}), (\{b\}, \{b\}),$
$(\{a,b\}, \{a,b\})\}$



Properties of Relations

Def^n. Let R $\subseteq A \times A$ be a relation defined on a set A.

Notation:
"aRb" means
$(a,b) \in R$

① If for all $x \in A$, xRx,
then we call R "reflexive".

e.g. $=$, same cardinality
$\subseteq$, $\leq$, $|$ (divides)
non-eg. $>$, $\neq$

② If for all $x, y \in A$, $xRy \Rightarrow yRx$,
then we call R "symmetric".

e.g. $=$, same cardinality, $\neq$
non-e.g. $>$, $<$, $\geq$, $\leq$, $|$

③ If for all $x, y, z \in A$,
$(xRy \wedge yRz) \Rightarrow xRz$,
then we call R "transitive"

e.g. $=$, $<$, $\subseteq$, $\leq$, $|$
"is some derivative of" on f^ns

non-eg. "is **the** (1st) derivative of" on f^n,
$\neq$, $\nmid$ (not divide)

Interesting example: R is $\neq$, $x=2$, $y=3$, $z=2$

$(2 \neq 3 \wedge 3 \neq 2)$ **BUT** $2 \neq 2$ is false