Proofs of Existence.

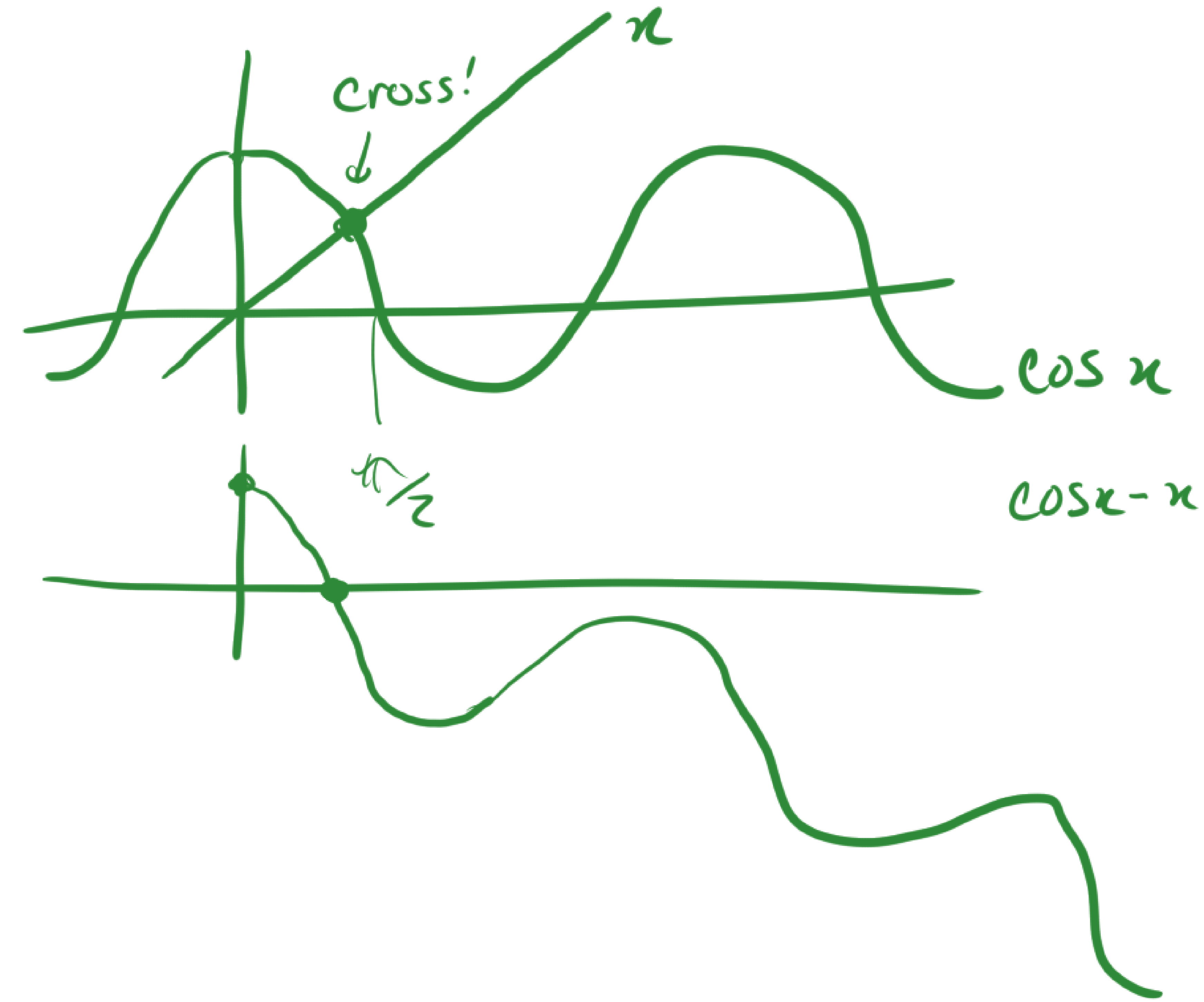
Ex. **Thm.** There exists a prime number p such that $p+2$ is also prime.

Pf. Let $p=3$. Then $p+2=5$, which is prime. $\square$

Thm. There exist integers m and n s.t. $2m+3n=12$.

Pf. Let $m=3$, $n=2$. Then $2m+3n=6+6=12$. $\square$

Thm. There exists $r\in\mathbb{R}$ s.t. $r=\cos r$.



Pf. Let $f(x)=\cos x - x$.

Then $f(0)=\cos 0 - 0 = 1-0 = 1 > 0$.

$f(\pi/2) = \cos(\pi/2) - \pi/2 = 0 - \pi/2 = -\pi/2 < 0$.

Since f is continuous, by the Intermediate Value Theorem, $\exists\ 0<r<\frac{\pi}{2}$ s.t. $f(r)=0$.

Then $\cos r = r$. $\square$

Thm. There exist irrational numbers a and b s.t. a^b is rational.

$$\sqrt{2}^{\sqrt{2}} \to \text{irrational?} \to \text{proof A of Thm.}$$

$$\sqrt{2}^{\sqrt{2}} \to \text{rational?} \to \text{proof B of Thm.}$$

$$\sqrt{2}^{\sqrt{2}} = 2^{\frac{1}{2}\sqrt{2}} = 2^{\frac{1}{\sqrt{2}}} = 2^{2^{-1/2}}$$

Irrational #s we know:

e, π,

proven. $\sqrt{2}$, $\sqrt{3}$

$\ln(2)$ ⇐ true but not proven.

$e^{\ln(2)} = 2$ ✓