Converse & Contrapositive

statement	contrapositive	converse
$P \Rightarrow Q$	$(\sim Q) \Rightarrow (\sim P)$	$Q \Rightarrow P$

(logically equivalent to original)

Statement: If $X \subseteq \mathbb{Z}$ then $3 \in X$ or $4 \in X$.

Contrapositive: If neither 3 nor 4 is in X then $X \not\subseteq \mathbb{Z}$. } log. equiv

Converse: If $3 \in X$ or $4 \in X$ then $X \subseteq \mathbb{Z}$. ← other.

Statement: If a function f is continuous then it is differentiable.

Converse: If a function f is differentiable then it is continuous.

Thm. $\sqrt{2}$ is irrational.

Pf. Suppose for a contradiction that $\sqrt{2} = \frac{a}{b}$ for some $a, b \in \mathbb{Z}$.

If a and b share a common factor, remove it. So a and b have no common factor.

Then $\sqrt{2}\,b = a$. So $2b^2 = a^2$

① We will show a is even.

By $a^2 = 2b^2$, a^2 is even.

So a is even. Write $a = 2c$, for some $c \in \mathbb{Z}$.

② We will show b is odd.

If b were even, then a and b would have a common factor.

③ We will show b is even.

Since $2b^2 = a^2$, we have $2b^2 = (2c)^2 = 4c^2$.

So $b^2 = 2c^2$.

So b^2 is even. So b is even.

We've shown b is both even and odd. This is a contradiction. ∎