Boolean Algebra

$P \wedge Q$

P	Q	P and Q
T	T	T
T	F	F
F	T	F
F	F	F

$P \vee Q$

P	Q	P or Q
T	T	T
T	F	T
F	T	T
F	F	F

$\sim P$

P	not P
T	F
F	T

"P is not true"

"negation"

Build big from small

logical formula

Ex.

P	Q	$P \wedge Q$	$\sim(P \wedge Q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

P	Q	$\sim P$	$\sim Q$	$(\sim P) \vee (\sim Q)$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

logically equivalent formulas

Analogy

	$\mathbb{Z}$	Boolean Algebra
elements	$\ldots, -3, -2, -1, 0, 1, 2, \ldots$	T, F
operations	$+, \times$	$\wedge, \vee, \sim$, others...
variables	$x, y, \ldots$	$P, Q, R, \ldots$
quantities	$3x + 7 + y$	$(P \vee Q) \wedge R$
equations	$3x + 7 = 17 + yx$	$P \wedge Q = P \vee Q$
true facts (for all inputs)	$3x + x = 4x$	$\sim(\sim P) = P$
algebraic manipulation rules	$(a+b)x = ax + bx$ "distributivity"	$\sim(P \wedge Q) = (\sim P) \vee (\sim Q)$ (De Morgan's Laws)

$P \Rightarrow Q$ P implies Q

P (beer)	Q (≥21)	if P then Q
T	T	T
T	F	F
→ F	T	T
F	F	T

P (inputs)	Q (inputs)	~P	~Q	(~Q) ⇒ (~P) (functions of inputs)
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

logically equivalent

$$P \Rightarrow Q \;=\; (\sim Q) \Rightarrow (\sim P)$$

ex. If you have a beer, you are ≥21.

If you are <21, then you don't have a beer.