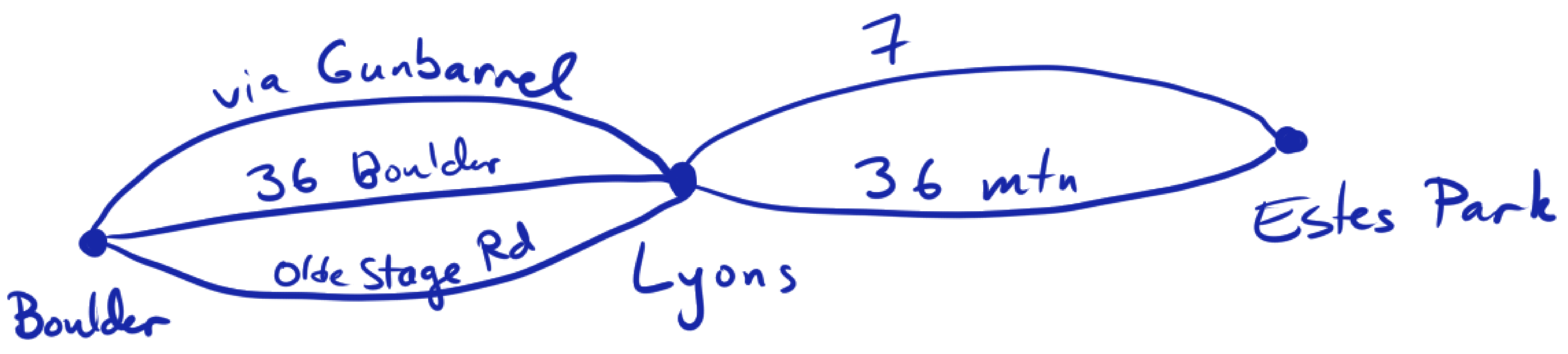
A counting question.



Q: How many ways to get from Boulder to Estes Park?

$$6 = 3 \cdot 2$$

$$= \left(\begin{array}{c}\#\text{ ways}\\ B \to L\end{array}\right) \cdot \left(\begin{array}{c}\#\text{ ways}\\ L \to E\end{array}\right)$$

One Viewpt: Process: $\left\{\begin{array}{lll} 1^{st} & \text{task} & (B \to L) \\ 2^{nd} & \text{task} & (L \to E)\end{array}\right.$

Another:

$$A = \{\text{via G}, \text{36 Boulder}, \text{Olde Stage}\} = \text{routes}_{B \to L}$$

$$B = \{7, \text{36 mtn}\} = \text{routes}_{L \to E}$$

$\text{Routes}_{B \to E}$

$$= A \times B$$

$$= \{(\text{36 Boulder}, \text{Hwy 7}), \ldots\}$$

(all combinations)

	36 mtn	7
via G		
36 Bouldr		
Olde St.		

$$6 = 3 \times 2.$$

Theorem / Fact $|A \times B| = |A| \cdot |B|$.

Multiplication Principle

To count the number of ways to accomplish a task which consists of 2 independent subtasks, we have

$$\#\text{ ways} = \left(\begin{array}{c}\#\text{ ways to}\\ \text{do task 1}\end{array}\right) \times \left(\begin{array}{c}\#\text{ ways to}\\ \text{do task 2}\end{array}\right)$$

independent is important.

it means no matter how one task is accomplished, the options for the other stay the same.

Power Sets

$$\boxed{\emptyset = \{\}}$$

Def^n. Let A be a set.

The <u>power set</u> of A, denoted by $\mathcal{P}(A)$

is

$$\mathcal{P}(A) := \{ X : X \subseteq A \}$$

In other words, the set of subsets of A.

Example. $A = \{1, 2, 3\}$

$$\mathcal{P}(A) = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$$

$$|\mathcal{P}(A)| = 8$$

$A = \emptyset$

$\mathcal{P}(A) = \{\emptyset\}$

$A = \{\emptyset\}$

$\mathcal{P}(A) = \{\emptyset, \{\emptyset\}\}$

$A = \mathbb{Z}$

$\mathcal{P}(A)$ includes things like

$\{1\}$, $\{2x : x \in \mathbb{Z}\}$ etc.

Counting Question

In our class of 32 students, how many ways can we form a committee? (no size restriction)

One viewpoint: A = set of students

$\mathcal{P}(A)$ = the set of possible comittees.

Then Q is $|\mathcal{P}(A)| = ?$

INDEPENDENT

Answer using multiplication principle.

1st task/person : yes or no (2 options)
(on comittee or not)

2nd task/person : yes or no (2 options)

⋮

32nd task/person : yes or no (2 options)

ways to form the committee is

$$\underbrace{2 \times 2 \times \cdots \times 2}_{32 \text{ times}} = 2^{32}.$$

Theorem. Let A be a finite set. Then $|\mathcal{P}(A)| = 2^{|A|}$.

Pf. We will use the multiplication principle.

Creating a subset of A can be divided into $|A|$ independent subtasks as follows:

Write $A = \{a_1, \dots, a_n\}$ where $n = |A|$.

The 1^{st} task is to decide if a_1 is in the subset. There are 2 ways to do this.

The 2^{nd} task - - - - - a_2 - - - -

- - - - - - - - -

⋮

In general, the j^{th} task is to decide if a_j is in the subset, and there are 2 ways to do this. (This is for $1 \le j \le n$.)

Since these tasks are independent, the # of ways to form a subset is

$$\underbrace{2 \times \cdots \times 2}_{n} = 2^n = 2^{|A|}.$$

Therefore the # of subsets of A is $2^{|A|}$, i.e.

$$|\mathcal{P}(A)| = 2^{|A|}.$$

∎

set of even integers $= \{ x \in \mathbb{Z} : x \text{ is even} \}$

$= \{ x \in \mathbb{Z} : x = 2k \text{ for some } k \in \mathbb{Z} \}$

$= \{ 2x : x \in \mathbb{Z} \}$

check ?

$\{ x \in \mathbb{Z} : 2x \}$