# Continued Fraction and Intro Coding Exercises – Math 4440

## Continued Fraction

Compute the continued fraction of 31/64:

$$\frac{31}{64} = \frac{1}{\frac{64}{31}} = \frac{1}{2 + \frac{2}{31}} = \frac{1}{2 + \frac{1}{\frac{31}{2}}} = \frac{1}{2 + \frac{1}{15 + \frac{1}{2}}}$$

At this point, we know we are done because all numerators are 1.

## Coding Theory

Consider the code $C = \{(0,0,1), (1,1,1), (1,0,0), (0,1,0)\}$.

1. What is the length of $C$?
2. What is $d(C)$ (the minimum distance)?
3. What are $q, n, M$ and $d$ if $C$ is a $q$-ary $(n, M, d)$-code?
4. What is the code rate of $C$?
5. How many errors can $C$ detect?
6. How many errors can $C$ correct?
7. Show that $C$ is not linear.
8. Suppose you send the codeword $(1,1,1)$ and 2 errors are made on the noisy channel, in the first and last positions. Explain what message is received and what it decodes to. Was communication successful?
9. Show that $C$ is a coset of a linear code. What is the linear code? Call it $C'$.
10. Find $n$ and $k$ so that $C'$ is an $[n,k]$-code.
11. Define $C'$ by linear equation(s).
12. Give a basis for $C'$.
13. Give a matrix for which $C'$ is the rowspace.

*Solutions.*

1. The length is 3 since there are 3 characters in each codeword.
2. We have to find the smallest distance between codewords. There are 6 pairs to check, and among these, we find that all pairs differ by 2. So $d(C) = 2$.

3. $C$ is a 2-ary (binary) $(3,4,2)$-code.
4. The code rate is
$$R = \frac{\log_2(4)}{3} = \frac{2}{3}.$$
5. $C$ can detect 1 error.
6. $C$ can correct 0 errors.
7. To show $C$ is not linear, we can find two codewords whose sum is not a codeword. For example,
$$(0,0,1) + (1,1,1) = (1,1,0) \notin C.$$
Alternatively, we can notice that $(0,0,0)$ is not in $C$, so it cannot be a subspace.
8. Suppose you send the codeword $(1,1,1)$ and 2 errors are made on the noisy channel, in the first and last positions. Then the message that is received is $(0,1,0)$, since first and last positions are "flipped". This message is a codeword, so it decodes to itself. The communication was not successful, since the decoded message isn't the message that was sent.
9. If we subtract $(0,0,1)$ from every element of $C$, we get
$$C' = \{(0,0,0),(1,1,0),(1,0,1),(0,1,1)\}.$$
Therefore $C = (0,0,1) + C'$. The code $C'$ is linear because it is a subspace of $\mathbb{F}_2^3$ (in other words, it contains 0 and is closed under addition; there are few enough elements to check exhaustively!).
10. $n = 3$ and $k = 2$. One way to see this is to give a basis for $C'$ which we do below. Another is to count: a $k$-dimensional subspace is of cardinality $2^k$. $C'$ it has 4 elements, so it is of cardinality $2^k$ for $k = 2$.
11. We expect one linear equation since it is of codimension 1 in the ambient vector space. The equation $x + y + z = 0$ seems to do it.
12. A basis is $(1,1,0)$, $(1,0,1)$. These are linearly independent. But actually any two non-zero vectors of $C'$ form a basis, so your answer may differ.
13. Just put the basis elements into a matrix as the rows:
$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$