

MATHEMATICS 4440/5440
CRYPTOSYSTEM ORGANIZER

Cryptosystem: *Affine Cipher*

Plaintext space: $(\mathbb{Z}/26\mathbb{Z})^{\text{str}}$

Ciphertext space: $(\mathbb{Z}/26\mathbb{Z})^{\text{str}}$

Key space (and its size): $(\mathbb{Z}/26\mathbb{Z})^* \times \mathbb{Z}/26\mathbb{Z}$ size = $12 \cdot 26 = 312$

Encryption:

$$\text{key} = (\alpha, \beta)$$

$$p \mapsto \alpha p + \beta \pmod{26}$$

Decryption:

$$\text{key} = (\alpha, \beta) \quad \text{figure out } \alpha^{-1}$$

$$c \mapsto \alpha^{-1}(c - \beta) \pmod{26}$$

Example:

$$= \alpha^{-1}c + (-\alpha^{-1}\beta)$$

↳ see notes

← decryption is encryption
w/ key = $(\alpha^{-1}, -\alpha^{-1}\beta)$
"decryption key"

Ciphertext only attacks:

exhaustive search (312 keys)

frequency analysis (permutation of histogram bars)

Known plaintext attacks: → see hmwk.

Chosen plaintext attacks: Choose plaintext $AB = (0, 1)$

$$0 \mapsto \alpha \cdot 0 + \beta = \beta$$

$$1 \mapsto \alpha \cdot 1 + \beta = \alpha + \beta$$

$$\beta = \text{ciphertext}(0)$$

$$\alpha = \text{ciphertext}(1) - \text{ciphertext}(0)$$

Chosen ciphertext attacks:

Choose $AB = (0, 1)$ as ciphertext.

Same analysis: get decryption key