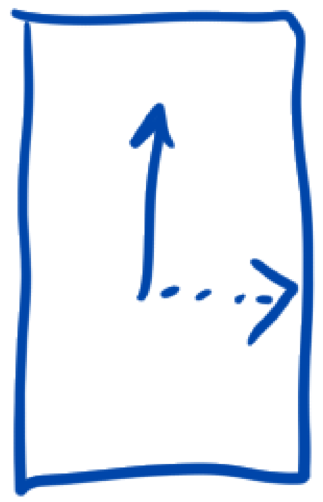
Example: Polarized Filters

photons = qubits

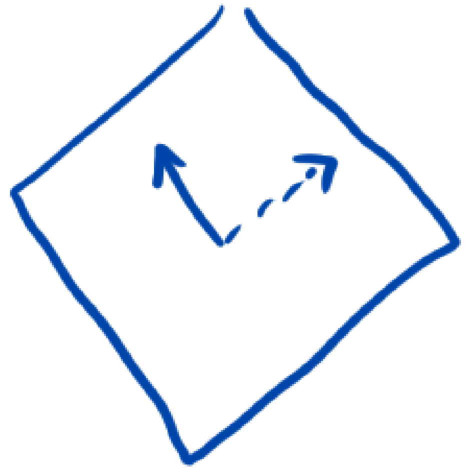
vertical polarization $|\uparrow\rangle$
horizontal polarization $|\rightarrow\rangle$
} a photon polarization is a superposition.

a filter is a measurement device



light passes = observed state $|\uparrow\rangle$

light blocked = " " $|\rightarrow\rangle$

$|\uparrow\rangle \longrightarrow$  $\longrightarrow$

$\xrightarrow{1/2}$ $|\nwarrow\rangle^{NW}$

$\xrightarrow{1/2}$ blocked $(|\nearrow\rangle)^{NE}$

$$\boxed{|\uparrow\rangle = \frac{1}{\sqrt{2}}|\nwarrow\rangle + \frac{1}{\sqrt{2}}|\nearrow\rangle}$$

Example: Polarized Filters

Experiment A

all polarizations

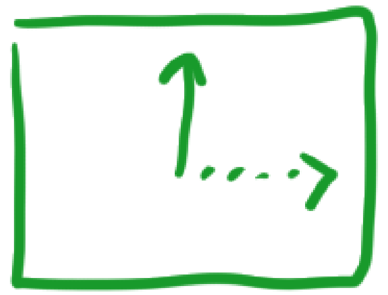
$\frac{1}{2}$ passes $|\uparrow\rangle$

$|\rightarrow\rangle$ blocked

$|\uparrow\rangle$ pass

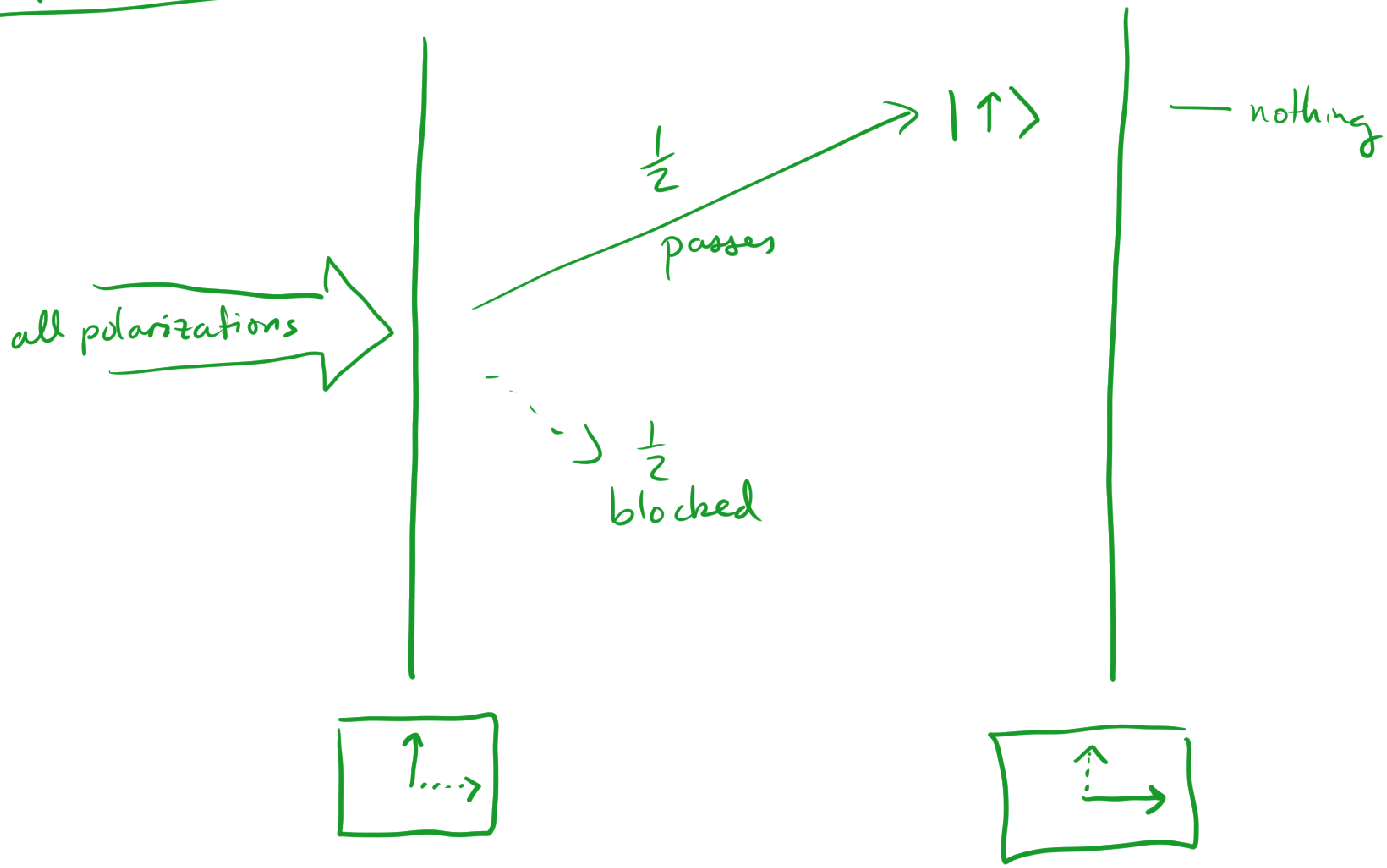
$\frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\rightarrow\rangle = |\nearrow\rangle$ 50-50

$\frac{1}{\sqrt{2}}|\uparrow\rangle - \frac{1}{\sqrt{2}}|\rightarrow\rangle = |\nwarrow\rangle$ 50-50



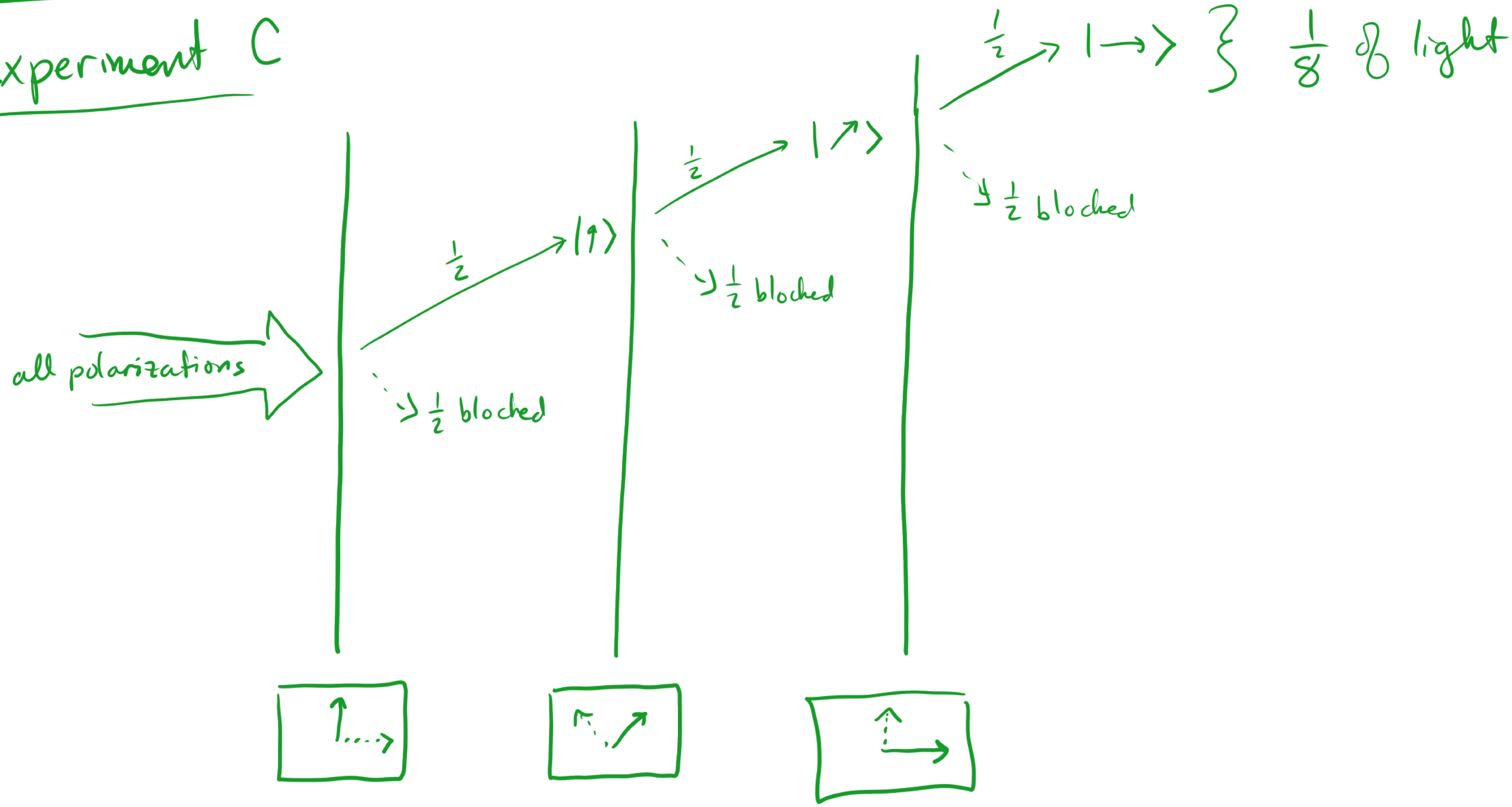
Example: Polarized Filters

Experiment B



Example: Polarized Filters

Experiment C



Quantum Key Distribution (BB84)

Bennett
Brassard

"L" basis	"V" basis
$\lvert\uparrow\rangle = 0$	$\lvert\nwarrow\rangle = 0$
$\lvert\rightarrow\rangle = 1$	$\lvert\nearrow\rangle = 1$

Alice / Bob

Bits	Bases	Photon	Bases	Measure	Bit
0	L	$\lvert\uparrow\rangle$	L	$\lvert\uparrow\rangle$	0
1	L	$\lvert\rightarrow\rangle$	V	random	
1	V	$\lvert\nearrow\rangle$	L	random	
0	V	$\lvert\nwarrow\rangle$	V	$\lvert\nwarrow\rangle$	0
0	L	$\lvert\uparrow\rangle$	L	$\lvert\uparrow\rangle$	0
1	V	$\lvert\nearrow\rangle$	V	$\lvert\nearrow\rangle$	1

Next, Alice & Bob reveal the bases. (classical channel)

Keep the bits where they agree as the secret key.

Quantum Key Distribution (BB84)

Attacks

If Eve measures a photon en route:

Eg. $|\uparrow\rangle$ in V basis: becomes $|\nwarrow\rangle$ or $|\nearrow\rangle$ (50-50 chance)

Then if Bob measures in L basis: 50-50 chance of $|\uparrow\rangle$ or $|\rightarrow\rangle$

$|\uparrow\rangle$ in L basis: no change, she's not detected.

So Eve has a $\frac{1}{4}$ chance of detection if the corresponding bit is revealed.

So: After comparing bases, 1 use some bits to check for Evesdropping (compare bits)

2 Keep rest for the secret key.

2 qubits

classical states: 00 01 10 11

quantum superposition:

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle.$$

w/ $\alpha_{ij} \in \mathbb{C}$, $|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$.