# Worksheet on Elementary Enumeration

September 9, 2015

1. Let's return to the counting problems we were working on last worksheet. How many different ways can we pick a committee from our class of 27 students? This time, let's assume a committee can have any number, from 0 to 27, of people. Committee members aren't ordered, so (Ted,Joe) is the same as (Joe,Ted), as a committee.
2. Now, suppose we want to pick a committee of zero people from the class. How many ways can we do that?
3. How many ways can we pick a committee of one person from the class?
4. How many ways can we pick a committee of two people from the class?
5. How many ways can we pick a committee of three people from the class?
6. How many ways can we pick a committee of four people from the class?
7. How many ways can we pick a committee of $n$ people from the class, if $0 \leq n \leq 27$? Write a nice formula for your answer using factorials. Make sure to justify your formula. Don't just guess it from the special cases we did. Give a general argument.
8. Now, use your answer from the last question to answer the question 1 of this section again, given in the form of summation notation.
9. Now, you've answered question 1 two ways. You have, in fact, proven a theorem. State the theorem you have proven. It should say that two different-looking formulas are equal.
10. If you answered the last question with a theorem about the number 27, state a more general one you can obtain by considering two answers to the question 'How many ways can you pick a committee from a room of $n$ people?' This is as much an exercise in notation as anything else.

# 1 More counting!

1. In the previous problems, you had some factorials divided by factorials. We can make a convenient notation:

$$\binom{n}{r} := \frac{n!}{r!(n-r)!} = \frac{(n)_r}{r!}$$

   The left equal sign is meant to signify a definition of the lefthand notation by the righthand quantity. Sometimes we use a colon in front of an equal sign to signify this. Check that the right equal sign is true, so that the quantity we are defining could be defined in either of the two ways.

2. Prove that

$$\binom{n}{r} = \binom{n}{n-r}.$$

   You could do this by looking at the definition and doing algebra, or by arguing that both sides actually count the same thing two different ways (hint: think about committees).

3. Prove that

$$\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}.$$

   by arguing that both sides actually count the same thing two different ways (hint: think about committees). It is much more unsatisfying to prove this by doing algebra on factorials.

# 2 Just to keep you busy

1. Suppose one puts $n$ points on a circle, and joins each pair by a line. Slide the points around a bit so that none of these line segments meet more than two at a time (i.e. no three meet at one point inside the circle).

   (a) How many lines are there?

   (b) How many regions is the circle cut up into? Hint: Count them via diagrams for at least 6 points before you make a conjecture.