Algebraic Number Theory Spring 2025

January 5, 2025

http://math.colorado.edu/∼kstange/teaching.html

1 Course Description

This will be a fairly standard course on algebraic number theory. I plan to use Matthew Baker's *Algebraic Number Theory Course Notes*, which covers the usual global theory material (standard highlights being Dirichlet's Unit Theorem and the finiteness of the class group), as well as an introduction to the local theory. You can find a link to these course notes on the course website above.

2 Pre-requisites

The course will be accessible to PhD students concurrently taking the algebra sequence who can invest a little extra time; we will play catch-up on Galois Theory as needed. I will assume knowledge of ring theory, including PIDs, UFDs and Euclidean domains as usually covered in *Graduate Algebra 1* here at CU. I will briefly review commutative algebra as it is needed, but not in the depth it may deserve (so take commutative algebra properly someday).

3 Resources

I include below a few good places to look for more information. There are hundreds of books on the subject, but I restrict myself to four you should probably know about:

1. Samuel, *Algebraic theory of numbers*. This is a wonderfully direct, concise introduction to the global theory.
2. Marcus, *Number Fields*. This is frequently used as a standard in this department (covering global theory only), perhaps because it is full of examples/exercises. The notation drives me batty.
3. Borevich and Shafarevich, *Number theory*. The first chapter is used as the standard for local theory in our department. This book is not just about *algebraic* number theory, and covers a wide variety of topics, including quadratic forms, local theory and analytic theory.
4. Neukirch, *Algebraic number theory*. This covers much more than a first course, both local and global, and takes a sophisticated approach from the first.

4 Credit

Homework will be assigned on a continuous basis as we cover material (approximately 1-2 problems per lecture), and every second Friday will be a presentation day, where students present and discuss the problems. Everyone registered for credit in the class will be called upon.

Students wishing to receive credit for the course shall attend lecture regularly, support a productive classroom atmosphere, and show evidence of work and understanding by having something competently prepared to present at least half the time they are called upon. Students wishing to receive an A for the course shall, in addition, complete the majority of the homework competently and thoughtfully, present most of the time they are called upon, and participate constructively in the discussion of other student presentations (it's ok to once in a while decline to present).

5 Recording

Depending on modality, I may take video recordings of lectures, as well as PDFs of in-class 'blackboard' notes. If so, these will be available to registered students and classroom guests on canvas (provided there are no technical difficulties). They will not be publicly available (for reasons of privacy for those attending).

6 Unexpected changes of format

I suffer from vestibular migraine. On rare occasions, I may be unable to attend class on short notice. On such days, I expect students to use the classroom hour to do homework problems together in a collaborative way, taking charge of themselves. In these cases, *class is not cancelled*. I will notify everyone via email.

7 Standard Syllabus Statements

All standard CU required syllabus statements apply to this class. See:
https://www.colorado.edu/academicaffairs/policies-customs-guidelines/required-syllabus-statements

In case of illness or religious observance, please email me and arrangements will be made to accommodate.