MATHEMATICS 152, FALL 2008
THE MATHEMATICS OF SYMMETRY

Contents

1. Specifics

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Email. `stange152@math.harvard.edu` This is a dedicated course email which won't get caught by spam filters and will show up in my inbox in bright red.

Cell phone. 401-241-9659. Emergencies only; please use email.

Course Assistant / Section. TBA.

Course Website. `http://isites.harvard.edu/k39821` Please make checking the course website, discussion boards and course blog part of your morning procrastination routine. **I will make announcements on the website and it is your responsibility to notice them.**

Course Meetings. 1:00-2:30 PM Tuesdays and Thursdays in Science Center 222

Textbook. "Discrete Mathematics," Norman L. Biggs, second edition, Oxford University Press, 2002, ISBN# 0-19-850717-8. At Harvard Coop.

Date: Last revised: September 7, 2008.

2. Goals and Prerequisites

This course is an exploration of the mathematics of symmetry. We will study the mathematics of the *discrete*. Roughly speaking, this is the mathematical realm where things are either equal or they are not: there is no measure of the "approximate." In particular, calculus, with its limits and Riemann sums, is irrelevant. We will cover groups, permutations, finite fields, finite geometry, graph theory, linear algebra, and probability. Our golden strand will be the symmetry group of the icosahedron, which will tie together all the disparate topics of the course. This is only one approach; we've chosen to follow this one voice through a complex and interconnected fugue.

The course also seeks to give students practice in mathematical communication. Students will be expected to present regularly to the class, to write proofs, and to participate actively in a course weblog maintained for the interested public. There will be reading assignments, homework assignments, blog post assignments, quizzes and exams. Students should practice discussing mathematics with their classmates, both during presentations and outside of class time on discussion boards and in sections, study groups and office hours. **Active participation is key.**

Prerequisites are at a minimum: 21b or equivalent is listed. Your calculus experience won't help specifically, but any math you have taken will give you a bit more facility with mathematical thought. Your experience with linear algebra will help. The primary prerequisite is a willingness to participate very actively and to work hard. It is also important to have a user's understanding of computers and the internet, as both of these will be used as tools for the course.

3. Seminar Style

This course is taught in seminar format. That means students will do the presenting of material in 5-10 minute blocks, assigned on a rotating basis. This will give everyone lots of practice in writing and presenting proofs and examples, etc. It also means your peers are counting on you to do a good job – they are learning from you! **I encourage you to come to my office hours to practice or discuss your presentations**, especially in the first few weeks of class.

You will receive a separate handout with tips on preparing for and giving presentations, as well as tips on how to approach mathematics in general.

4. Interacting with the Course

Instructor's Office Hours. You have many choices:

- **Regular Office Hours:** Tuesday 10-11 am, Wednesday 1:30-3 pm. Guaranteed and preferred gold club points eligible.
- **Virtual Office Hours:** The course will have discussion boards and/or an email list (details pending), where I will answer questions.

If you send me email directly I may post the question anonymously on the discussion board or list along with an answer. (If you don't want that to happen, just say so.)

- **Your Very Own Office Hours:** You can always make a private appointment.

CA Office Hours/Section. TBA.

Email List and/or Discussion Boards. The course will use an email list and/or discussion boards to communicate (details pending). These can be used to discuss the course itself, homework problems, etc.

Study Groups. Please take a moment now, while reading this syllabus, to get the names and contact info of the people nearest you in the class. It will be to your advantage to locate those in your dorm and hold study sessions.

Name	Contact Info
.....	.....
.....	.....
.....	.....
.....	.....

Polygons. You'll receive polygon kits and make polygons with coloured edges to be used during the course. We'll have a polygon-making party at the beginning of year to get to know your classmates and eat things (TBA).

5. Grading

The grading breakdown will be as follows:

Final Exam	100
Best Quiz	30
Second Best Quiz	30
Worst Quiz	15
Required Homework	50
Exploratory Homework / Program	28
Class Presentations	20
Reading Project	15
Blog Postings	12

Extra credit is possible from casino night or other projects to be decided during the course. The total points available, ignoring extra credit, are thus 300. The grading scheme is as follows:

Percentage	Minimum Grade
93%	A
87%	A-
81%	B+
75%	B
69%	B-
63%	C+
57%	C
50%	C-

6. Quizzes and Exams

Exams: There will be three quizzes during the semester. They will each be designed to last approximately one hour, but you will be given 1 hr 50 min in which to complete them. There is a 'repeat' date for each quiz, on which you can attempt to achieve a better score (although the quiz will change). **The maximum score for a repeat quiz is 80%, so this is only worth it if you scored less than 80% on the original.** But you only get one chance on the final!

Three Quizzes: **Wednesday, October 8th,**
7-8:50 pm, Yenching Auditorium
repeated Wednesday, October 15th,
7-8:50 pm, CGIS-South S-001 (80% maximum)
Wednesday, November 5th,
7-8:50 pm, Yenching Auditorium
repeated Wednesday, November 12th,
7-8:50 pm, CGIS-South S-001 (80% maximum)
Wednesday, December 3rd,
7-8:50 pm, Yenching Auditorium
repeated Wednesday, December 10th,
7-8:50 pm, CGIS-South S-001 (80% maximum)

Final Exam: **Wednesday, January 14th** (Exam group 15,16)

7. Homework Assignments

There are two kinds of homework problems: **required** and **exploratory**.

Required Homework. Homework will be due at the start of each Thursday's class, and returned one week later. You are encouraged to discuss the course with other students, the course assistant, and the instructor, *but you should always write your homework solutions out yourself in your own words.* Doing the required homework is one of the most important parts of the course.

Exploratory Homeworks. Exploratory problems are 2 points each and will consist of more difficult proofs or open-ended problems. They are designed to engage your creativity and make you play around with mathematics.

Over the course you must achieve a total of 28 points in exploratory homeworks (14 problems). Due dates for the exploratory problems are flexible, but to get full credit you must earn

- at least 6 points by Tuesday, October 7th
- at least 12 points by Tuesday, November 4th
- at least 18 points by Tuesday, December 2nd
- the full 28 points before the final exam

Doing exploratory problems can be a good way to review for quizzes and tests, but do not leave them for the last minute, since you will do a better job if you take time to do them slowly.

8. Computing Project Option

For those interested in programming, you can substitute one programming project for 6 exploratory homeworks (i.e. 12 points). Warning: the programming project option may take more time, especially if you are inexperienced. However, if you are concentrating in Computer Science or Applied Mathematics, this may be a good choice. In these projects (there are three to choose from), you implement key mathematical ideas from the course in interactive Web pages using PHP. You need only follow step-by-step instructions to create the user interface, but you will need programming experience in a language like C or Java (CS 50 or AP Computer Science) to implement the mathematics. PHP is an easy language to learn, so no previous experience with it is necessary. You can see what the finished projects will look like by following the link under Programming Projects on the course Web site.

If you like to program but have no interest in learning how to create Web pages, you may, for 6 points, just implement the mathematics in programs with no user interface at all.

All the necessary files for installing PHP on your own Windows computer in order to do the programming projects can be found on the course Web site.

Paul Bamberg, the faculty member who designed these projects, has offered to help students who choose to do them.

You can do up to 12 points in computing project options. (One full PHP project or two math-side-only projects).

9. Reading Assignment

During the course, you are expected to choose an article from the *American Mathematical Monthly* magazine (or similar magazine) relating to the topics studied in the course, read it, and write a review about it, aimed at

giving other students a brief description of the article: its purpose, interest, results, methods and consequences. As a rough guide to length, try to aim for approximately 1000 words, but nobody is a stickler; quality is far more important than quantity. The review should contain actual mathematics, and the audience for this is your peers in the class (so you can assume familiarity with all the material studied in the course). For most articles, it will be appropriate to:

- explain the motivation for the question(s); explain the relationship to Math 152; explain why they are interesting to the author, and why they are interesting to you (these may differ)
- give some background information on the problem (you may want to do some research beyond the article if it seems appropriate)
- state the main theorem(s) (if there are several you may choose to concentrate on one or two) – this is the one place where it is okay to quote the article directly: cite it by giving the Theorem number used in the article and copy the statement exactly as written
- explain what the statement(s) of the theorem(s) means: explain any notation, terminology or definitions not used in class, with examples if possible (your own examples are best)
- give an interesting example or two of the result in practice – you should, if possible, write out an example of your own, not copy one from the article
- a brief description of tools/methods used in the proof or in the theory in general – understand the proof in the article and then write a brief overview of it in your own words
- describe any interesting consequences or corollaries – do you notice any the author hasn't mentioned?

I do not in general expect you to give full proofs from the article, although if you find the method of proof (or any other aspect of the article) interesting, you can discuss it in more detail.

The idea is most emphatically **not** to simply copy the most important sentences directly from the article, or write a concise version of the full article. *The review should be written in your own words, emphasising, summarising and exploring what you've learned from the article. It is not just a condensed rewriting!* If you aren't clear on what I'm looking for, if you have any other questions or if you just want to brainstorm about it, talk to me.

The finished product will be posted on the weblog (but don't worry about the 'educated laymen' reading the blog), so you can do it as text with embedded LaTeX (so it appears as a blog post) or as a pdf file produced from any editor such as Word or LaTeX, with a link in a blog post.

You will find suggested articles on the course website. I wish to avoid repetition. Therefore, when you decide you wish to use one of these, kindly email me and I will mark it as belonging to you. Do not begin your review

until you hear confirmation from me, since if I have several requests for the same article, I will make an arbitrary decision.

Deadlines for this project will be listed in the Deadlines section of the website. You must sign up for a deadline of your choice from among those available. They are spread out through the second half of the semester so that your classmates will have time to read them, and I will have more time to help with them. Those who go earlier have less of the course as background to use, but have more articles to choose from and will complete the project before the end-of-term crunch.

The project will be judged on overall writing style, on whether the mathematics is correct and clearly exposited, and whether the student demonstrates full understanding of the article, and whether the student takes the material further than just a re-explanation (come up with examples, extensions, consequences and reactions to the content).

10. Math 152 Weblog

Math 152 will maintain a public blog about the mathematics in the course (and outside the course) and the experience of the course itself. The intended audience is the educated lay public, and the purpose of the blog (for the students) is to provide experience in mathematical exposition and communication, make connections between mathematics and the wider world, encourage the students to interact with each other, and solidify course knowledge. Students will contribute to the blog according to both assigned posting projects and also whenever the fancy strikes them.

Possible topics. Why the plot in Numb3rs makes no sense, where you saw a symmetry group on the street, why you love/hate the most recent topic in the course, tips and tricks for making superb paper polyhedra, a math article you read, a math joke or a great math blog that you found, your favourite recipe for pi, a probability brainteaser, etc. etc. Sometimes the assignment will be to explain a single concept from class. You could choose to do a "week wrap-up" mentioning what topics we've covered and what caught students' (or your) interest. Sometimes it will be to look up an outside reading topic and give a brief description of its relationship to the course (these will often be the most fun). There are many other possibilities.

Posts can be long or short, or even just an interesting link, and comments are encouraged. The only rule is that the topic must relate to mathematics (any mathematics), in some way. Oh, and common decency applies, of course: be polite and respectful (and respect the online privacy of other students). Students can choose to have their public online identity anonymous (but known to myself and the class).

Web Address. `http://math152.wordpress.com`

Student Accounts. To get started, please get yourself a wordpress account at http://wordpress.com, and fill out your profile as much as you like. You can give yourself an alias and hide your real name if you like. You can also use a wordpress account that you already have, but I request that on whichever account you are using, you fill out certain mathematics-related information in your profile (details below). Therefore, depending on your preferences as to your online social life and privacy etc., you may wish to open a dedicated account for this course. You will use the account to post to the Math 152 blog. As soon as you have made an account or decided on one to use, email me with the account username or the email address you are using for it, so I can add you to the blog.

In your profile, you should include the following information (you can be creative in the format and add anything else you deem relevant): Your concentration, reason for taking the course, relevant interests and background in mathematics, anything unusual about you as a student for the course and finally, take a few minutes to tell us: what is your relationship to mathematics? I appreciate thoughtful answers, and if you feel you would like to tell me some of these things in private, then please feel free to email me a more detailed answer (please still include the basics on your profile). *I really am interested!*

Privacy Concerns. The weblog is a public webpage. While much of the course webpage and discussion forums etc. are for students and teaching staff only, the weblog is viewable by all and can be found in search engines. However, students have a right to privacy, and can choose to post using an anonymous account (as mentioned above). The identity of the student will, however, be known to myself and the other students. Therefore, students should not reveal the identity of another student online.

Additionally, to post online, students must enter the text of the post into an online form. Postings can be saved as drafts and/or marked private. If it is marked private, no one but the user can see it, *even when it is published.* If it is not marked private, the weblog administrator (myself) can see it in draft form, and everyone can see it when published. However, I promise not to read student drafts. Therefore, you don't need to use the 'private' option, but should you decide to do so, *don't forget to uncheck it when you publish.*

Help using Wordpress. The Weblog is hosted by Wordpress. I can provide help in using Wordpress, but the best resource is Wordpress' own help pages (click 'help' on the website), which will explain all the features of Wordpress. If you need a new email address to sign up for a Wordpress account (you can only have one account per email address), it is easy to obtain free additional email addresses through Gmail, Yahoo, Hotmail etc. I recommend Gmail, as it is very easy to use and does not attract very much spam.

Math on Wordpress. One of the (many) reasons I've chosen to host the course blog on Wordpress.com is that it supports LaTeX, a mathematical typsetting language. Click on 'LaTeX' on the blog for a one-minute tutorial on how to display equations and matrices in a blog post. It's very simple.

Blogging Assignments. There are two types of blogging assignments:

- **Regular blog topics.** As the course progresses, I will post suggested topics for blog posts on the main website. Dates for posts will be assigned in a rotating manner. You can choose from the pool of topics when your turn arrives, or invent your own. Once registration settles, I will assign each student 6 dates spread throughout the semester. Posts will be worth 2 points each: one point for making a mathematically-relevant post on the correct day (you can write them early and postdate) and one point for making an effort, and having correct mathematics (if you are unsure about anything, you can email me or see me before posting).
- **Reading Assignments.** Please see the section on reading assignments, below. These, when finished, will be posted on the blog.

You are also expected to read the blog, of course!

11. Casino Night

At the end of semester we will hold Casino Night, with mock money, but real rewards. The top three players will receive 3,2,1 bonus points respectively. Players will start with a loan of a certain number of chips, and play blackjack, roulette and craps, etc. Players who dress up will receive a larger loan, according to the schedule below. The winners will be determined on **winnings**, not on capital, i.e. on the amount left after the initial loan is payed back. Therefore dressing up for a bigger loan doesn't automatically move you up through the standings, but may keep you in the game longer.

- Mathematical Costume: Your peers will vote on how much you deserve as your loan: from \$350-\$600.
- Star: An outfit closely matching any picture of a movie star or famous person in a (real or movie-set) casino (you must bring the picture for comparison and your peers will vote on whether it passes). \$500 loan.
- Dressy: Tuxedo or full suit and tie, or cocktail dress or ball gown. \$450 loan.
- Made an effort: Nice enough not to get kicked out of a fancy restaurant. \$400 loan.
- Regular clothing: anything else. \$350 loan.

Please arrive on time.

12. Approximate Unofficial Estimated Day-by-Day Syllabus

THE OFFICIAL VERSION IS ON COURSE WEBSITE

Date		Reading in Biggs	Topics
Sept	16		Intro, Counting, Symmetries, Permutations
	18		Intro, Counting, Symmetries, Permutations
	23	1.1-6, 3.1-5	O#1: Proof
	25	5.1-4, 10.6, 12.5-6	O#2: Permutations
	30	20.1-3, 21.1	O#3: Groups
Oct	2	8.4, 13.1-3	O#4: Congruence Arithmetic
	7	20.4-7	O#5: Subgroups, Cosets, Quotient Groups
	8		**QUIZ #1 7 pm: O#0-4**
	9	20.8-9	O#5: Subgroups, Cosets, Quotient Groups
	14	Ch. 22, 23?	O#6: Rings and Fields
	15		**QUIZ #1 Repeat 7 pm**
	16	Ch. 22, 23?	O#6: Rings and Fields
	23	supplement	O#7: Finite Affine Geometry
	28	supplement	O#7: Finite Affine Geometry
	30	supplement	O#7: Finite Affine Geometry
Nov	4	supplement	O#8: Linear Algebra
	5		**QUIZ #2 7 pm: O#5-7**
	6	supplement	O#8: Linear Algebra
	12		**QUIZ #2 Repeat 7 pm**
	13	supplement	O#9: Group Isomorphisms
	18	supplement	O#9: Group Isomorphisms
	20	supplement	O#10: Sets and Probability
	25	supplement	O#10: Sets and Probability
Dec	2	supplement	O#10: Sets and Probability
	3		**QUIZ #3 7 pm: O#8-10a**
	4		O#10: Sets and Probability
	9		O#11: Graphs and Groups
	10		**QUIZ #3 Repeat 7 pm**
	11		O#11: Graphs and Groups
	16		O#11: Graphs and Groups
	16		**Casino Night! 8-10 pm (tentatively scheduled)**
January	14		final examination