Name:

Lecture Section (Professor or hour):

October 22, 2002

Math 9, Quiz #6

Question 1 *Using implicit differentiation, find* $\frac{dy}{dx}$*:*

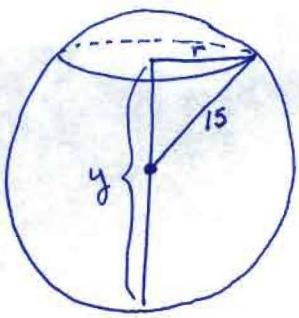
$1 = y^2 + y\ cos(e^{x^2})$

$$0 = 2y\frac{dy}{dx} + \frac{dy}{dx}\cos(e^{x^2}) + y(-\sin(e^{x^2}))e^{x^2}(2x)$$

$$\frac{dy}{dx}(2y + \cos(e^{x^2})) = y\sin(e^{x^2})e^{x^2}\cdot 2x$$

$$\frac{dy}{dx} = \frac{2yx\,e^{x^2}\sin(e^{x^2})}{2y + \cos(e^{x^2})}$$

Question 2 *As you learn calculus, your brain, which is a fluid (at least after so much calculus it is), drains out of your head, which is spherical. Suppose your head has a radius of 15 cm. Suppose that right now, the brain-fluid is at a depth of 27 cm in your brain. Suppose that right now, the rate of change of the radius of the surface of the fluid is +2 cm/sec. What is the rate of change of the depth of the fluid right now? (You may leave your answer as a fraction.)*



From the triangle, $15^2 = (y-15)^2 + r^2$.

Differentiating implicitly,

$$0 = 2(y-15)\frac{dy}{dt} + 2r\frac{dr}{dt}.$$

We know: $y = 27$, $\frac{dr}{dt} = 2$ We want: $\frac{dy}{dt} = ?$

So we still need: r ?

But, $15^2 = (y-15)^2 + r^2$

so $15^2 = (27-15)^2 + r^2$

so $15^2 = 12^2 + r^2$

so $r^2 = 15^2 - 12^2$

so $r = 9$

Plugging in, $0 = 2(12)\frac{dy}{dt} + 2\cdot 9\cdot 2$

$$\Rightarrow \frac{dy}{dt} = -\frac{36}{24} = -\frac{3}{2}.$$