

MATHEMATICS 152, FALL 2008
THE MATHEMATICS OF SYMMETRY
Outline #1 (Proof)

Last modified: September 4, 2008

Reading. Biggs, Sections 1.1-6, 3.1-5. Reading Biggs means working some exercises as you go, just for practice. Also be sure to read and think about the exercises in Section 1.2, which have interesting discussions (the ‘discussions’ are not solutions).

In this class we won’t bother with the logical notations such as \neg and \wedge which Biggs uses in Chapter 3 (the notation \implies for ‘implies’ is nice though). Nor will we bother with truth tables. However, he uses this language in the text, so you will have to get used to it there in order to read Chapter 3.

1. In proper formal mathematical language, write clearly and present
 - The definition of a prime number.
 - A theorem about prime numbers of the form “The number n is not a prime number.” (That is, you pick an n .)
 - A proof of that theorem.
 - A theorem about prime numbers of the form “The number n is a prime number.”
 - A proof of that theorem.

(Biggs 1.6)

2.
 - Explain the terms *universal statement* and *existential statement*. Illustrate each with three or four examples, some of which are true and some of which are false.
 - Explain the term *counter-example* and give some counter-examples to the false universal statement(s) you gave in the first part.
 - Give a statement which could be considered both a universal and an existential statement.

(Biggs 1.4-5)

3. Explain the concept of negation. (Don’t bother with Biggs’ notations \neg etc., but you can just write “NOT(statement)”) What is the negation of ‘A and B’? What is the negation of ‘A or B’? What is the negation of ‘B and not A’? Explain why. (Biggs 3.1-2)

4. Consider the statement “If I am from mars, then you are from venus.” Is this true or false? Explain. (Biggs 3.3)
5. Explain and give examples of the concepts of ‘contrapositive’, ‘converse’ and ‘if and only if’ (don’t bother with Biggs’ notations \neg etc.) Explain the relationship between a statement and its contrapositive. Explain the relationship between a statement and its converse. (Biggs 3.4-5)

Consider the statement ‘a perfect square which is odd must be the square of an odd number.’

- Put the statement in ‘if - then’ theorem form. (Hint: You can begin with a hypothesis before the ‘if - then’ part of the theorem if you like.)
 - Prove the statement by stating its contrapositive and proving that.
 - State and prove the converse of the statement.
 - State a theorem using the phrase ‘if and only if’ that combines your two results.
6. Give the proof in Exercise 1.6.4 in Biggs and help the class discover what is wrong with it.
 7. Give the following Definitions and Theorems (we can assume these theorems are true):
 - **Definition.** A *flog* is a greeb that has positive appoplactation.
 - **Definition.** A *gawaxian* is a flobbert that has inert feebles.
 - **Theorem.** Any ablutareen is either a greeb or a flobbert.
 - **Theorem.** Any ablutareen with inert feebles has positive appoplactation.

Now prove the following theorem.

Theorem. Any ablutareen with inert feebles is a flog or a gawaxian.

8. Explain what proof by contradiction means. Show that 8 is not a perfect square by proof by contradiction.