**Some motivational problems in number theory**

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For each problem, indicate whether you think the answer is YES or NO. Then indicate how hard you think it is: (E)asy, (M)edium, (H)ard, (U)nsolved.

0.1. **Are there infinitely many primes?**

0.2. **Is there a closed formula for the $n$-th prime?** Let's say a function of $n$ a calculus student could read.

0.3. **Is there a (possibly multivariate) polynomial that gives exactly all the primes when evaluated on all integer inputs?**

0.4. **Are there infinitely many primes of the form $4n+1$?**

0.5. **If you know the $n$th prime ends in a particular digit, is the final digit for the $(n+1)$-st prime equally likely to be any of the four possibilities $1, 3, 7, 9$?**

0.6. **Are there infinitely many primes of the form $n^2+1$?**

0.7. **Are there infinitely many primes $p$ for which $p+2$ is also prime?**

0.8. **Up to any bound $N$, are there always more natural numbers with an odd number of prime factors than with an even number of prime factors?**

0.9. **Does $x^2 - 1141y^2 = 1$ have any integer solutions?** Note: if you ask the computer to check up to 25 digits, it will find none.

0.10. **Does $x^3 - y^2 = 1$ have any integer solutions besides $(1, 0)$?** Note: if you ask the computer to check up to 25 digits, it will find none.

0.11. **Does $x^n + y^n = z^n$ have any integer solutions for integers $n > 2$?** Note: if you ask the computer to check up to 25 digits, it will find none.

0.12. **Is there an algorithm to determine if a given polynomial equation in any number of variables has an integer solution?**

0.13. **Are there any quadratic forms with integer coefficients which represent all positive integers?** A quadratic form is a degree two polynomial whose monomials are individually total degree two, for example $x^2 + 7y^2$.

0.14. **Does there exist a polynomial-time algorithm to determine if $n$ is prime?** 'Polynomial time' means the time taken (or number of basic logical operations needed) grows polynomially in terms of the number of digits of $n$.)

0.15. **Can we factor numbers in deterministic polynomial time?**