

MATHEMATICS 2001
FALL 2015
INTRODUCTION TO DISCRETE MATHEMATICS

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WHAT YOU NEED TO KNOW

- (1) This course is about mathematical thinking and proof.
- (2) Content is secondary.
- (3) You will be absolutely expected to do daily homework including readings.
- (4) We will not lecture all material.
- (5) In-class time will frequently be spent interactively in groupwork.
- (6) You will be assessed only in-class (quizzes and worksheets), and the final exam.
- (7) Assessment in-class will be *standards-based*. In short, you will earn levels of accomplishment after repeated attempts.

LARGE SCALE COURSE GOALS

First, to become proficient in the culture and practice of theoretical mathematical thinking; to:

- (1) understand written mathematics,
- (2) communicate mathematics,
- (3) study mathematics effectively in an independent way,
- (4) distinguish logical correctness and error,
- (5) cultivate mathematical creativity,
- (6) synthesize and use novel definitions,
- (7) create novel proofs,

and, I hope, to enjoy the process.

Second, to become proficient in the basic use of:

- (1) logic,
- (2) set theory,
- (3) functions,
- (4) induction,
- (5) counting,
- (6) further topics (potentially number theory, game theory, graph theory).

A Reflection on Goals. These goals are complementary. The so-called ‘content’ of the course actually serves two purposes: material on which to practice the first goal, and useful knowledge that will be called upon in later mathematics courses. So it is essential to the course, but even *more* essential is to learn the ‘theoretical’ way of thinking.

SCHEDULE

A rough guess as to how semester may go:

- Week 1: Modern Mathematics and the Art of Reading
- Week 2: Sets
- Week 3: Logic
- Week 4: Counting
- Week 5: Direct Proof
- Week 6: Contrapositive Proof
- Week 7: Proof by Contradiction
- Week 8: Proving Non-Conditional Statements
- Week 9: Proofs Involving Sets
- Week 10: Disproof
- Week 11: Mathematical Induction
- Week 12: Relations
- Week 13: Functions
- Week 14: Cardinality
- Week 15: Review

GRADING

Your grade will be the better of:

- 70% semester grade and 30% traditional final
- 50% semester grade and 50% traditional final

I will use a fixed grading scheme:

- A: 90 % and above
- A-: 80% and above
- B+: 75% and above
- B: 70 % and above
- B-: 65% and above
- C+: 60% and above
- C: 57% and above
- C-: 54% and above
- D+: 52% and above
- D: 50% and above
- F: anything else

(There is no D- grade)

SEMESTER GRADE

- Reading 5%. This will be assessed by reading assignments.
- Writing 25%. This will be assessed by proof writing.
- Reasoning 25%. This will be assessed by proof writing.
- Synthesis 5%. This will be assessed by proof writing.
- Content 40%. This will be assessed with short-answer questions.

All assessments will be in-class, and they will occur daily. Each day will vary a little, but it will consist of learning and assessment. Learning will be lecture, groupwork, clickers, or other activities. Assessment will be quizzes or handing in work performed during class. Generally, individual definitions or concepts will see numerous opportunities to assess, with the best scores counting.

ASSESSMENTS

Reading assignments: This will be in-class work. You will work together to read some text. At the end of class there will be an individual quiz to assess your understanding. This will happen on at least three separate days; your highest score will count. Here are some of the specific skills involved in reading mathematics:

- (1) Create an example and counterexample when reading a definition.
- (2) Create an example satisfying a statement.
- (3) Create a counterexample to a statement (if one exists).
- (4) Test the loosening or tightening of a hypothesis in a statement.
- (5) Identify a misunderstanding or trace an argument by use of examples.

- (6) Identify and verify the required hypotheses for a logical step.
- (7) Locate logical errors.
- (8) Pinpoint misunderstanding.

Proof Writing: This will happen throughout the semester. You will write proofs, sometimes in a quiz context, sometimes over the course of a class-period as groupwork. These will be graded for Writing, Reasoning and (sometimes) Synthesis. The top 7 samples of your Writing, Reasoning and Synthesis grades (including at most three of them from the first half of semester), will be averaged to obtain your grade. If you do not provide sufficient evidence for a meaningful grade, your effort on any given proof may not be graded at all (for example, if you miss the main idea and erroneously create a very short proof with insufficient writing to evaluate). Attached to this syllabus is a Gradesheet rubric.

Short answer: This will be short-answer quizzes that occur frequently. For each concept, if you get a problem on that concept correct, you will earn that ‘badge’ and you are done that concept. The score will be based on the total number of badges earned out of those available. There will be many (but not infinite) opportunities to attempt badges again. Attached to this syllabus is a list of badges for the first two chapters.

FINAL EXAM

There will be a traditional final exam, which is very similar to the in-class assessments, meaning the above three types. Please note that on the final there will be no opportunities to ‘re-assess’, so you must be prepared to answer questions on any topic in the class for your final grade.

Final Exam: Thursday, December 17th, 7:30 pm - 10 pm

WORKING AT HOME

You will be assigned home learning tasks between each lecture. These tasks are not for a grade, but *if you are not doing them, you will not succeed.*

We only have 3 hours together in class, so you must learn to work independently. The course has no major milestones such as midterms or projects, so the courseload should be even throughout semester. Expect to work between each class.

The Boulder Faculty Assembly at the university has made a motion (BFA-X-M-9-0105) which specifies that “**An undergraduate student should expect to spend approximately 3 hours per week outside of class for each credit hour earned.**” I will try to respect this workload.

MATERIALS

Textbook. *Book of Proof*, 2nd Edition, Richard Hammack.

This textbook is available free online at
<http://www.people.vcu.edu/~rhammack/BookOfProof>.

Clickers. You must purchase a clicker if you do not already have one.

Calculator is not required.

DOING MATH: INTERACTING AND FINDING HELP

Virtual Office Hours: You may email math and course questions to
kstange@math.colorado.edu.

Questions of interest to your classmates will be anonymized and answered by the instructor on D2L, for the benefit of all students, unless you request otherwise.

Instructor’s Office Hours:

- **Regular Office Hours:** TBA, 2 hrs per week.
- **Floating Office Hour:** For students who cannot make the regular office hours, I will announce an extra office hour fitting the schedule of those that ask; this hour may change each week and will be posted online and open to all.
- **Private Office Hour:** You can always make a private appointment. This is only for private matters.

Discussion Boards. Are available on D2L, where you can write in LaTeX (math symbols; click ‘Advanced’ and investigate the editors). I **strongly** recommend discussion of any questions you may have.

Study Groups. Please take a moment now, while reading this syllabus, to get the names and contact info of the people nearest you in the class. It will be to your advantage to locate those in your dorm and hold study sessions.

Name	Contact Info
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CHEATING

If you fail to follow the rules below, you are in violation of the Honor Code of the University of Colorado. Failure to comply may result in a course grade of zero.

- (1) During tests, you may not use any resources besides your brain and your pencil. No phones, no books, no internet, no nothing.

- (2) When handing in collaborative in-class work, you must name the students in your group.

MISSED OR LATE WORK

Missing the final exam is a major problem and will be addressed via university policies.

Missing in-class work is at your discretion. Please stay home if you are sick. All the assessment in this class is given many times and only your best performances matter, so there will be no make-ups for specific missed work.

SPECIAL REQUESTS

I am happy to accommodate disabilities or religious observances, or a request that I address you with a different name or pronoun than my roster indicates. Please contact me as soon as possible.

It's possible you suffer from math anxiety. Although fear of math is like fear of chocolate, even mathematicians can suffer from it! Please come talk to me.

UNIVERSITY POLICIES

Please see the course website for University Policies concerning such matters as religious holidays, the Honour Code, harassment, etc.

SAMPLE BADGES

These goals will be assessed individually and repeatedly, and badges for each can be ‘earned’. There are half-badges for attempts demonstrating a partial understanding. They will be assessed with short-answer questions. They are classified (purely for convenience) as:

- (1) Definition (correct use of notation, evaluating whether something satisfies the definition, and generation of examples), and
- (2) Concept (a more nebulous idea, demonstrating an understanding of a concept, such as a proof method or a single proof, by explaining it well or generating correct inferences, examples, etc).

A small number of the concepts are more advanced and will require independent learning (i.e. we will not concentrate on them in class).

0.1. **Sets.** 15 badges.

0.1.1. *Definition.*

- (1) Definition: set, element, equality of sets, empty set, cardinality of set
- (2) Definition: set-builder notation and \mathbb{Z} , \mathbb{N} , \mathbb{R} , \mathbb{Q} , interval notation
- (3) Definition: ordered pair, ordered n -tuple, Cartesian products and powers
- (4) Definition: subset, power set
- (5) Definition: union, intersection, difference
- (6) Definition: universe, complement, Venn diagram
- (7) Definition: infinite sums/intersections/unions, index set
- (8) Definition: division algorithm

0.1.2. *Concepts.*

- (1) Concept: cardinality of Cartesian product or power
- (2) Concept: order of operations and nonsensical notation
- (3) Concept: counting subsets
- (4) Concept: counting by independent choices
- (5) Concept: well-ordering principle
- (6) Concept: numbers defined as sets
- (7) Concept: Russell’s paradox

0.2. **Logic.** 15 badges.

0.2.1. *Definitions.*

- (1) Definition: statements and truth value, open sentences
- (2) Definition: and/or/not/implies/iff and their symbols
- (3) Definition: truth tables of and/or/not/implies/iff
- (4) Definition: converse
- (5) Definition: logical equivalence
- (6) Definition: universal and existential quantifiers
- (7) Definition: logical inference

0.2.2. *Concepts.*

- (1) Concept: computing truth tables of logical statements, and using them to verify truth or logical equivalence
- (2) Concept: recognizing the logical structure of english sentences
- (3) Concept: DeMorgan's Laws, contrapositive, commutative, distributive, associative
- (4) Concept: using quantifiers and symbols to write statements
- (5) Concept: order of quantifiers
- (6) Concept: quantifying open statements
- (7) Concept: translating english to logical symbols
- (8) Concept: negating sentences, particularly with quantifiers

MATH 2001 PROOF GRADESHEET

0.3. **Writing.** Grade: 0 1 2 3 4 ungraded

This is the art of writing mathematics **for an audience**. Areas that need improvement:

- (1) Complete and simple sentences, appropriately sized.
- (2) Do not include extraneous information.
- (3) Keep structure in line with logical steps.
- (4) State assumptions.
- (5) Introduce variables appropriately.
- (6) Guide the reader.
- (7) Choose notation to maximize clarity.
- (8) Identify the use of hypotheses.
- (9) Keep structure organized on the page and legible.
- (10) Precision over vagueness.
- (11) Honesty about logical gaps or imprecision.
- (12) Value simplicity.
- (13) Observe the established culture/etiquette.
- (14) Do multiple drafts as needed.

0.4. **Logical Reasoning.** Grade: 0 1 2 3 4 ungraded

This is the art of correct and logical reasoning from hypothesis to conclusion. Areas that need improvement:

- (1) Justify logical steps.
- (2) Choose appropriately sized logical steps.
- (3) Identify logical holes in an/your argument precisely.
- (4) Identify hidden assumptions.
- (5) Choose the fastest or clearest route.
- (6) Do not include extraneous reasoning.
- (7) Avoid arithmetic errors.
- (8) Correct use of contrapositive or contradiction.
- (9) Do not forget cases.

0.5. **Synthesis.** Grade: 0 1 2 ungraded

This is the art of combining, extending and adapting previous experience to novel problems. For this proof, the type of synthesis needed was:

- (1) Combine two methods in sequence.
- (2) Work with a novel definition in terms of known definitions.
- (3) Invent a new method by analogy to an old one.
- (4) Adjust a method to a new context.
- (5) Draw conclusions from the combination of known statements.
- (6) Choose appropriate concepts for a given context.
- (7) Recognize a known mathematical structure in a new context.