Reading: Definitions and Questions

August 26, 2015

1 Reading Definitions

Things to do when reading a definition.

1. **Give some examples of things that satisfy the definition.**

   (a) Explain how they satisfy the definition (i.e. verify the definition). Sometimes this involves some type of computation or check. For example, suppose you read 'n is odd if it is of the form $2k+1$ for some integer k'. Then to verify that 9 is odd, you must write $9 = 2 \cdot 4 + 1$, i.e. $k = 4$.

   (b) Give some outlandish ones. Things you might be tempted to think don't satisfy the definition. For example, the set of my left foot is a set, even though my left foot is a weird thing to think about. This clarifies that sets can be made up of any kind of element at all, not just numbers.

2. **Give some examples of things that do not satisfy the definition.**

   (a) Explain why each one fails. What part of the definition does it fail to satisfy?

   (b) Try to come up with as many different 'types' of failure as possible, and explain each type. For example, does it fail because it is not the right type of object to begin with (my socks fail all the definitions in this section because they are not numbers!)? Or maybe it fails because of a technical detail later in the definition.

   (c) Try to come up with different ones than the book suggests for each 'type,' if possible.

   (d) The best 'non-examples' are the ones that are *almost* examples, except for one little problem. Try, as much as possible, to give these, as they can be particularly helpful to understanding. For example, my socks are not a very good non-example, because they are non-examples to pretty much everything in the book, so we don't learn a lot from them. And no one was going to think my socks were prime,

anyway. But -5 is really *almost* a prime number, and someone might think it is. So that's a helpful non-example.

3. **Give an 'intuitive' explanation of the definition.**

   A sentence or two you might explain to your dog, or that helps you remember the main idea.

4. **List any potential 'gotchas'.**

   These may be confusions you had for a moment but cleared up for yourself. For example, special cases (1 or 0?), counterintuitive terminology, unexpected restrictions on the type of object (e.g. "remember this definition doesn't apply to negative numbers!"), etc. Pretty much anthing you might remind your friend of while tutoring him in this. (You may not think of anything for particularly simple definitions, but there's usually something!)

2 Questions

Asking good questions is an art.

1. A good question is one whose answer will clarify your understanding.
2. The point is NOT just to ask hard questions. The point is to ask questions that are important to clarify at this juncture. They can be simple questions, or more complicated ones.
3. A good way to approach this is to imagine teaching the subject. What pitfalls should you warn your students away from, or clarify before they get confused?
4. A question may be answerable by you, or maybe the author has not given enough information.

Here are some example good questions for Sets, Section 1.1. The author does a good job of answering some of these as you continue reading, but you should be asking before you know whether he will answer them.

1. *Do the elements of a set need to be numbers?*
2. *Can sets have repeated elements?*
3. *Is the set $\mathbb{Z}$ infinite?*
4. *What's an example of two sets of the same size that aren't equal?*
5. *Can I come up with an example of two sets that are equal but of different sizes?*
6. *What are the elements of $\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$?*