Quiz on Comps, Parts, Perms, Gen-fns etc.

October 9, 2015

Problem 1 Grade: /2.5 writing, /2.5 logic&math. Take as given the following true fact about the Stirling numbers of the second kind, for all $x \in \mathbb{R}$ and $n \in \mathbb{Z}^{>0}$:

$$x^n = \sum_{k=0}^{n} S(n,k)(x)_k = S(n,0)+S(n,1)x+S(n,2)x(x-1)+S(n,3)x(x-1)(x-2)+\cdots+S(n,n)x(x-1)\cdots(x-n+1).$$

Use this to find a closed formula for $S(n,3)$ in terms of $n \geq 3$. Present your solution in the form of a theorem and proof. *Hint: plug in some of your favourite numbers and see what happens.*

Problem 2 Grade: /2.5 writing, /2.5 logic&math. Let K_n be a sequence defined by $K_0 = K_1 = K_2 = 1$ and $K_n = K_{n-2} + K_{n-3} - 1$ for $n \geq 3$. Find a closed form **for the generating function** for this sequence (you do not need to simplify, once you have a closed form). Present your solution in the form of a theorem and proof.

Problem 3 Short answers. Provide justification, but succinctly.

1. (3 points) Compute $c(4,3)$ (this is the signless $(4,3)$-Stirling number of the first kind). Provide a justification.

2. (3 points) Explain why the number of partitions of n into exactly k parts is equal to the number of partitions of n such that the largest part is of size k.

3. (4 points) How many functions are there $f : [n] \to [k]$ whose image misses exactly one point? Provide a justification.