

1. (10) Let

$$A = \begin{bmatrix} 1 & 5 & -2 & 0 \\ -3 & 1 & 9 & -5 \\ 4 & -8 & -1 & 7 \end{bmatrix}, \quad \mathbf{p} = \begin{bmatrix} 3 \\ -2 \\ 0 \\ -4 \end{bmatrix}, \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} -7 \\ 9 \\ 0 \end{bmatrix}.$$

Use the fact that $A\mathbf{p} = \mathbf{b}$ to express the vector \mathbf{b} as a linear combination of the columns of A .

2. (20) Determine whether the columns of the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$$

span \mathbb{R}^3 . (Explain your answer.)

3. (20) Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form, where

$$A = \begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

4. (20) Determine whether the following sets of vectors are linearly independent or linearly dependent, giving a short explanation in each case. (Hint: you may be able to do this problem without doing any calculations.)

$$(i) \left\{ \begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 9 \end{bmatrix} \right\};$$

$$(ii) \left\{ \begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 9 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix} \right\};$$

$$(iii) \left\{ \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 8 \end{bmatrix} \right\};$$

$$(iv) \left\{ \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 8 \end{bmatrix} \right\}.$$

5. (30)

(i) Show that the map $S : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ given by

$$S(x_1, x_2, x_3, x_4) = (1, x_1 + x_2, x_2 + x_3, x_3 + x_4)$$

is not linear.

(ii) Show that the map $T : \mathbb{R}^2 \rightarrow \mathbb{R}^4$ given by

$$T(x_1, x_2) = (2x_2 - 3x_1, x_1 - 4x_2, 0, x_2)$$

is linear, by finding a matrix that implements the mapping.

- (iii) Determine whether the map T above is (a) injective (“one-to-one” in the book) and/or (b) surjective (“onto”). (Hint: you can answer this without doing any row operations.)

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Mathematics 3130: First In-Class Exam

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Problem	Points	Score
1	10	
2	20	
3	20	
4	20	
5	30	
Total	100	