

My Homework

My Name

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Question 1.2.3.4.5.6.7

Your text goes here. Here's an example displayed equation:

$$\mathcal{S} = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z} \right\}$$

and here's another:

$$3 \equiv 7 \pmod{4}.$$

Displayed equations display nicely centered on their own lines. Look up symbols you don't know at detexify.kirelabs.org.

You can also write inline equation like this: $a = bc$. They fit right into the text. I can make fractions like $\frac{1}{3}$ and I can say something is an integer like this: $z \in \mathbb{Z}$.

Here's an example theorem.

Theorem 1. *There are infinitely many primes.*

Proof. The number 2 is certainly prime (it is divisible only by 1 and itself), so there is at least one prime.

Suppose, for a contradiction, that there are only finitely many, say n of them, and list them as follows:

$$p_1, \dots, p_n.$$

Then consider the integer

$$N = p_1 p_2 \cdots p_n + 1.$$

N has a remainder of 1 when divided by any of the p_i . Therefore it is not divisible by any of the p_i . But it is certainly greater than 1 and hence divisible by some prime, which must not be in our finite list. By this contradiction, the theorem is proved. \square

Remark: This proof depends on the fact that every number is divisible by some prime. We haven't proven that yet.

An exercise in the notion of divisibility

Here's a definition from your textbook.

Definition 1. *Let $a, b \in \mathbb{Z}$. We say that a is divisible by b (and write $b \mid a$), if there exists an integer c such that $bc = a$.*

Do the following exercise:

Exercise 1. *Here's a statement of a potential theorem: "Let $a, b \in \mathbb{Z}$. If $a \mid b$ and $b \mid a$, then $a = b$." This theorem is actually (slightly?) false. Try to prove it and in doing so, discover the problem and correct it. \LaTeX corrected theorem and proof, written very carefully. You'll be judged (informally) by how nicely \LaTeX 'd it is, and on your exposition. Be neither too brief, nor too wordy. Don't include extraneous information, but don't just include equations, either. Please do use scrap notes before you start \LaTeX 'ing.*