Rubric for Grading Student Solutions

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In a proof, we seek:

1. First, a complete and correct logical proof of the assertion.
2. Second, a readable exposition.

Students will be assigned to grade individual problems. Each one will be graded out of four points. Most problems consist of a single proof, so this rubric is tailored to that situation.

Points assigned

0 points. The problem was not completed, or what was handed in has no or almost no bearing on the problem. Simply stating relevant definitions or theorems is not generally worth any points (although it is a reasonable start to doing the problem). In some clases, students are encouraged to write anything for partial credit, but if the solution has no helpful logical steps toward a solution, it should receive a zero.

1 point. There is at least some sign of relevant logical steps. For example, a relevant theorem was chosen and there is at least some indication of how it should be applied (i.e. what objects are expected to satisfy the hypotheses). There is, however, no indication of the outline of the full solution, and most logical steps are missing. There is less than half of a complete correct solution present. This grade is also appropriate in cases that the student has misunderstood the question or made an unwarranted simplifying assumption that results in a trivial or near-trivial solution, even if that solution is correct.

2 points. There is evidence of a reasonable basic proof outline, and several correct logical steps can be identified. However, writing may be unclear and there are major holes in the argument. In the aggregate, such holes will require lengthy repair (more than just a few sentences). Anywhere from a quarter to three quarters of a correct argument may be present in a solution that receives 2/4 credit, depending on other factors. If the proof is substantially correct but very confusingly written, a grade of 3/4 is more likely appropriate. A grade of 2/4 should usually indicate that substantial mathematics is missing.

3 points. The proof provided is an approximation of a full, complete proof. It has at least some important deficiencies, however. Its deficiencies may include substantially confusing writing, readily fixable logical errors or holes *that could have been filled in* without too much extra work. Examples of logical errors or holes include ruling out trivial cases (e.g. the case that a denominator is zero!), forgetting to check hypotheses of applied theorems (where doing so requires some small work, but they do in fact hold), or making unwarranted simplifying assumptions that can be avoided without too much extra work. If such an error or omission requires a lengthy repair, a grade of 2/4 is more appropriate. For example, assuming a group is finite for no reason may allow a slightly simplified proof, or a vastly easier proof. In the former case, the student may receive 3/4. In the latter case, they should receive less (assuming the proof for the finite case is correct). A complete and logically correct proof that is incredibly confusingly written may receive a grade of 3/4 for poor exposition.

4 points. The proof is decently written (does not need to be perfect, and may have typos or other errors that do not bear significantly on understanding), and it is logically complete and correct. There are no important steps missing or assumed. There are no unwarranted assumptions or forgotten special cases. The suggestions you may have for improvement all come under the category of 'improvements for clarity' rather than 'correcting logical errors.' Tiny omissions may also be ignored, if they can reasonably be assumed to be 'obvious' (e.g. pointing out that the integers are commutative). With regards to omissions, the proof should be judged in context of the material. For example, it may be important to state each ring axiom used when verifying basic ring properties from the axioms, but more suitable to omit such details when we've moved on to more advanced material.

Further remarks

How to decide. In practice, here is how I grade:

1. Read the proof, writing comments in detail.

2. Evaluate how much of a complete argument it is, and assign a tentative grade based on that, or decide that it is borderline between two grades.

3. Consider the quality of writing and increase or decrease the grade (especially if it is borderline) based on that.

The grader's responsibility in the case of poor writing. Notice that the quality of writing influences the grade in two ways: first, a poorly written solution is likely to be misunderstood and receive a lower grade because it is believed to be logically less complete; and second, explicitly as part of the grading decision. If a solution is written so poorly that the grader cannot see the correct logic in it, then it does deserve the lower grade. On the other hand, it is the grader's responsibility to make a good faith effort to see through the fog to the correct mathematics (and if this is challenging, to potentially decrease the grade on account of poor exposition).

Bonus points. Occasionally a student invents a novel or particularly clever or elegant solution. A bonus point may be appropriate in this situation.

Small errors with big consequences. Grading is based on how much of a complete logical argument there is. If a small but vastly simplifying error is made early on, the rest of the proof may be correct but much too easy; in this case a grade as low as 1/4 may be warranted (or even 0/4 in some cases), even if the underlying error is small or an easy one to make, simply because the solution becomes three lines long instead of a page of hard work. This is unfair in a certain way, but other resolutions would be unfair in others. We will apply this viewpoint consistently.

More is not better. Although the grading is losely based on the idea of 'how much of the proof' there is, more is not better when it comes to the character count. A good solution should be one that can be read easily. This means that each sentence has a coherent purpose, the proof does not meander, notation is kept to the essentials, and only appropriate details are included. Pronouns should not be used unless antecedents are abundantly clear (equation numbering and judicious introduction of notation are appropriate alternatives). Fluff should be removed (e.g. 'and thus we are done'). An excellent solution is often among the shortest in the pile, but all the essentials are included.

Comments. Write comments justifying your grade. Point out exactly where the logical hole is, and why. If possible, *give a counterexample* to the incorrect inference. Write constructive criticism of exposition (e.g. explain that a certain wording confused you and suggest alternatives). You may refer to this rubric.