Enumeration

September 17, 2015

1. How many ways can you colour the complete graph with n colours?
2. How many compositions of n are there?
3. How many compositions of n into k parts?
4. How many weak compositions into k parts?
5. How many weak compositions?
6. Give a combinatorial proof:

$$\binom{n+m}{k} = \sum_{i=0}^{k} \binom{n}{i}\binom{m}{k-i}$$

7. Give a combinatorial proof:

$$\sum_{m=k}^{n} \binom{m}{k} = \binom{n+1}{k+1}.$$

8. Give a combinatorial proof that for $0 < k \leq n$,

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

where $S(n,k)$ is a Stirling number of the second kind (which counts the number of partitions of $[n]$ into k non-empty blocks).

9. Let $|X| = n$ and $|Y| = m$
   (a) How many functions are there $f : X \to Y$?
   (b) How many injective functions are there $f : X \to Y$?
   (c) How many bijective functions?
   (d) How many surjective functions?
10. Give a combinatorial proof:

$$B(n+1) = \sum_{i=0}^{n} \binom{n}{i} B(i).$$

Here, $B(n)$ is the Bell number, the number of partitions of $[n]$ into non-empty blocks.

11. Suppose one puts n points on a circle, and joins each pair by a line. Slide the points around a bit so that none of these line segments meet more than two at a time (i.e. no three meet at one point inside the circle).
    (a) How many lines are there?
    (b) How many regions is the circle cut up into? Hint: Count them via diagrams for at least 6 points before you make a conjecture.