

# Quiz Proof #3

September 16, 2015

## Tools

You may call on these definitions and you may use the propositions below without having to prove them.

**Definition 1.** *An integer  $b$  is said to be divisible by another integer  $a$  (written  $a \mid b$ ) if  $b = ak$  for some integer  $k$ .*

**Definition 2.** *An integer  $n > 1$  is called prime if it has only two positive divisors, namely 1 and  $n$ . An integer  $n > 1$  is called composite if it is divisible by some integer  $b$  satisfying  $1 < b < n$ .*

**Definition 3.** *An integer  $n$  is called even if it is divisible by two. Otherwise it is called odd.*

**Proposition 1.** *If  $n$  is an even integer, then  $n + 1$  is an odd integer.*

**Proposition 2.** *If  $n$  is an odd integer, then  $n + 1$  is an even integer.*

## Task

**Theorem 1.** *If  $p$  is a prime number greater than 2, then  $p + 1$  is composite.*

Write a proof. This will involve several logical pieces, not just one. Please be careful to fully verify *all* the conditions of the definitions you use. *Hint: First show that  $p$  is odd. Then what do we know about  $p + 1$ ?*