Chapter 3 and 4 Review

Review your homework problems and their solutions first! These are the most helpful problems. Then, you can do problems from this review booklet, or some in your text (for the text problems, I have suggested interesting ones from the higher numbers – it is a good idea to first do low-numbered problems similar to any you didn't do correctly on the homework, so I will let you choose those yourselves):

Good Luck!!

Section 3.1: 30 - 36
Section 3.2: 36-37, 43, 45
Section 3.3: 32, 36, 37, 38
Section 3.4: 43-47, 61-62, 67-68
Chapter 3 True/False: 13, 39 (false), 41 (if true, give example), 45, 48
Section 4.1: 39-41
Section 4.2: 63-64
Section 4.3: 61, 62, 64
Chapter 4 True/False: 1, 2, 3, 18, 28, 36, 51

1. Let S be the subspace of P_2 of polynomials $f(x)$ such that $f(2) = 0$.
   (a) Show that S is in fact a linear subspace of $P_2[x]$.
   (b) Find a basis for S; you have to show that what you found is indeed a basis.

2. Consider the basis $B_1 = \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$ of $\mathbb{R}^2$. Let T be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ and $A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}_{B_1}$ be the matrix representing T in the basis B_1; that is

$$T \begin{bmatrix} z \\ w \end{bmatrix}_{B_1} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}_{B_1} \begin{bmatrix} z \\ w \end{bmatrix}_{B_1}.$$

Find the matrix representation of T in the standard basis.

3. Let $T : \mathbb{R}^5 \to \mathbb{R}^4$ be the linear transformation $T(\vec{v}) = A\vec{v}$, where

$$A = \begin{bmatrix} 1 & 4 & 2 & -5 & 1 \\ -1 & -3 & -1 & 1 & 0 \\ 4 & 13 & 5 & -8 & 1 \\ 3 & 7 & 1 & 5 & -1 \end{bmatrix}.$$

   (a) Find a basis for the image of T.
   (b) What is the dimension of the kernel of T?

4. Let P_2 denote the linear space of all polynomials of degree ≤ 2. Let $T : P_2 \to P_2$ be the linear transformation $T(f) = f + f'$. Is T invertible? If yes, find its inverse.

5. Let $\mathfrak{B} = (\vec{v_1}, \vec{v_2})$, where $\vec{v_1} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\vec{v_2} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$.

   (a) Show that $\mathfrak{B}$ is a basis for $\mathbb{R}^2$. What are the $\mathfrak{B}$-coordinates of the vector $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$?
   (b) Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation with $\mathfrak{B}$-matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. What is the standard matrix representing T in cartesian coordinates?

6. Let
$$A = \begin{bmatrix} -1 & -2 & 0 & 1 \\ 2 & 4 & -1 & 3 \\ -3 & -6 & 2 & -6 \end{bmatrix}.$$
Find a basis for the kernel and the image of A.

7. Let P_2 denote the linear space of polynomials of degree ≤ 2. Let $T : P_2 \to \mathbb{R}^3$ be the linear transformation
$$T(f) = \begin{bmatrix} f(-2) \\ f(-1) \\ f(2) \end{bmatrix}.$$
Show that T is invertible and find $f \in P_2$ such that $T(f) = \begin{bmatrix} -4 \\ 3 \\ 12 \end{bmatrix}$.

8. (a) Let
$$P = \frac{1}{27} \begin{bmatrix} 18 & -12 & -3 & -3 \\ -12 & 8 & 2 & 2 \\ -3 & 2 & 14 & -13 \\ -3 & 2 & -13 & 14 \end{bmatrix}.$$
Assume that P is the projection matrix onto a subspace W of $\mathbb{R}^4$. Find a basis for W.

9. (a) Let T be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by
$$T\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 - x_3 \\ -x_2 \\ x_1 + 7x_3 \end{bmatrix}$$
Find the matrix A representing T with respect to the basis
$$\vec{v_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{v_2} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{v_3} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

(b) Assume that

$$A = \frac{1}{420}\begin{bmatrix} 648 & -7306 & 6320 & -2154 \\ -360 & 2445 & -960 & 765 \\ 468 & -1446 & 1500 & -774 \\ 936 & -8317 & 6080 & -2493 \end{bmatrix}.$$

Let $\vec{v_1}, \ldots, \vec{v_4}$ be a basis for $\mathbb{R}^4$ with

$$\vec{v_1} = \begin{bmatrix} 9 \\ 0 \\ 3 \\ 8 \end{bmatrix}, \quad \vec{v_2} = \begin{bmatrix} 5 \\ -3 \\ 0 \\ 7 \end{bmatrix}, \quad \vec{v_3} = \begin{bmatrix} 1 \\ 1 \\ 5 \\ 3 \end{bmatrix}, \quad \vec{v_4} = \begin{bmatrix} 4 \\ 1 \\ 2 \\ -1 \end{bmatrix},$$

and let $M = [\vec{v_1}|\vec{v_2}|\vec{v_3}|\vec{v_4}]$. Assume that

$$\begin{aligned} A\vec{v_1} &= 2\vec{v_1}, \\ A\vec{v_2} &= \vec{v_1} + 3\vec{v_2}, \\ A\vec{v_3} &= 5\vec{v_1} - \vec{v_3}, \\ A\vec{v_4} &= \vec{v_1} + 2\vec{v_2} + \vec{v_3} + \vec{v_4}. \end{aligned}$$

Find $B = M^{-1}AM$. *Hint: very little equation is required.*

10. Let M_2 denote the vector space of all 2×2 matrices and $B = \begin{bmatrix} 2 & 6 \\ 0 & 3 \end{bmatrix}$.

(a) Let T be the linear transformation from M_2 to M_2 defined by $T(C) = B^{-1}CB$. Consider the basis for M_2 consisting of

$$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, E_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, E_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, E_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

The 4×4 matrix $A = \begin{bmatrix} 1 & & & 0 \\ 3 & & & -3 \\ 0 & & & 0 \\ 0 & & & 1 \end{bmatrix}$ represents T with respect to the basis $E_1, \ldots, E_4$. Supply the entries in the 2nd and 3rd columns of A.

11. (a) Let V be the subspace of $\mathbb{R}^3$ spanned by $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$. Find the matrix of the projection onto V.

(b) Let V be as in part (a) and $(\vec{b}) = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$. Find the projection of $\vec{b}$ onto V.

(c) Let V and $\vec{b}$ be as in parts (a) and (b). Let W be the subspace of $\mathbb{R}^3$ spanned by $\vec{b}$ and the vectors in V. Find the matrix of the projection onto W. *Hint: this requires almost no computation. Think!*

12. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 1 & -1 & 1 \\ 2 & 4 & 1 & -2 & 1 \\ -3 & -6 & -2 & 3 & -2 \end{bmatrix}$. The reduced row echelon form of A is $\begin{bmatrix} 1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Find a basis for the image of A. Determine the dimension of this space and the $\mathbb{R}^n$ in which it is contained.

(b) Find a basis for the kernel of A. Determine the dimension of this space and the $\mathbb{R}^m$ in which it is contained.

13. The vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -4 \\ 2 \end{bmatrix}$ are linearly independent. Find a basis for $\mathbb{R}^3$ which contains $\vec{v}_1$ and $\vec{v}_2$ as two of its three elements.

14. Consider the two bases for $\mathbb{R}^2$, $B_1 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$ and $B_2 = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$. Write and expression for the change-of-basis matrix from B_1 to B_2.

15. Suppose $\vec{v}_1$ and $\vec{v}_2$ are two linearly independent vectors in $\mathbb{R}^2$. Find the change-of-basis matrix from $B_1 = \{\vec{v}_1, \vec{v}_2\}$ to $B_2 = \{\vec{v}_2, 2\vec{v}_1\}$.

16. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation given by

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \\ x-y \end{bmatrix}.$$

Write an expression for the matrix representation of T in the bases $B = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$ of $\mathbb{R}^2$ and $B' = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$ of $\mathbb{R}^3$.

17. Consider the linear transformation $S : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$S\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+2y \\ x-y \end{bmatrix}.$$

Suppose $M = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ is the matrix representation of S in the coordinates of some basis B of $\mathbb{R}^2$ to the standard basis. Find the basis B.

18. True or false?

(a) The vectors $\begin{bmatrix} 1 \\ 3 \\ -5 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$ form a basis of $\mathbb{R}^3$.

19. Does $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} \right\}$ span $\mathbb{R}^3$?

20. Find a matrix A which transforms the vectors $\vec{v}_1 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$ and $\vec{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ into the standard coordinate vectors $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\vec{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ in $\mathbb{R}^3$. In other words, $A\vec{v}_i = \vec{e}_i$ for $i = 1, 2, 3$.

21. Let V be the vector space consisting of all polynomials $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ of degree ≤ 3. Let T be the linear transformation

$$T\left(p(x)\right) = x^2\,\frac{d^2p}{dx^2} + p(x).$$

   (a) Find the matrix A associated to T for some suitable basis of V.

22. Determine whether the vectors

$$\vec{v}_1 = \begin{bmatrix} 0 \\ 2 \\ 0 \\ -2 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 5 \\ 3 \\ -5 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 2 \\ -7 \\ 6 \\ 4 \end{bmatrix}, \vec{v}_4 = \begin{bmatrix} -1 \\ 3 \\ 2 \\ -2 \end{bmatrix}$$

span $\mathbb{R}^4$. Are they linearly independent?

23. Let

$$A = \begin{bmatrix} 1 & 3 & 1 & 3 \\ 2 & 1 & -3 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & 2 & 2 & 1 \end{bmatrix}.$$

   (a) Find a basis for the kernel of A.

   (b) Are the columns of A linearly independent? If not, find the linear dependence relation.

24. Let

$$A = \begin{bmatrix} 1 & 3 & -1 & -2 \\ 2 & 4 & -1 & -1 \\ -3 & -9 & h & 6 \end{bmatrix}.$$

For which values of h, if any, is the transformation $T(\vec{x}) = A\vec{x}$ onto? For which values of h, if any, is T $1-1$?

25. Suppose $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation with

$$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_1 + x_2 \\ 3x_1 + 3x_2 + x_3 \\ x_1 - x_2 \end{bmatrix}.$$

Find the matrix A of the transformation T.

26. (a) Suppose that $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation with $T(\vec{e}_1) = \vec{e}_1$ and $T(\vec{e}_2) = \vec{e}_1 + \vec{e}_2$. Give a possible value for $T(\vec{e}_3)$ so that T is invertible. Explain why your answer works.

27. Check which of the following subsets are vector subspaces:

(a) The set of all vectors $\vec{u} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ in $\mathbb{R}^3$ such that $x_1 = 2x_3 - x_2 + 1$ and $x_1 + 3x_2 = 0$.

(b) The set of all 2×2 matrices with determinant equal to 0.

28. Check which of the following functions are linear transformations:

(a) The function $F : \mathbb{R}^4 \to \mathbb{R}^3$ with $F\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_4 \\ 3x_1 + x_3 \\ x_4 - 1 \end{bmatrix}$

(b) The function $F(A) = A^T - 5A$ defined on the space of 4×4 matrices.

(c) The function $F : P_7 \to P_7$ with $F(p(t)) = 2p'(t) + p(0)$.

29. (a) Find bases for the subspaces Kernel(A) and Image(A) associated to the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 15 \\ 3 & 14 & 25 & -3 \end{bmatrix}.$$

30. (a) Let H be the subspace of P_3 spanned by

$$\{1 + 2t + t^3, t - t^2, -3 + 2t - 8t^2 - 3t^3, -1 - 2t - t^3\}.$$

Does the polynomial $2 + 4t + 2t^2 + 2t^3$ belong to the above subspace H?

(b) Find a basis for H.

31. (a) Show that $\mathcal{B} = \left\{ \vec{v}_1 = \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ -6 \\ 7 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$, and find the matrix of the change of coordinates from $\mathcal{B}$ to the standard basis of $\mathbb{R}^3$.

(b) Find the matrix of the change of coordinates from the standard basis of $\mathbb{R}^3$ to $\mathcal{B}$.

32. Determine whether or not the following sets of vectors are linearly independent. If they are linearly dependent, express v_3 as a linear combination of v_1 and v_2.

(a) $\vec{v}_1 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$.

(b) $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1 \\ 4 \\ 9 \end{bmatrix}$.

33. Consider the matrix $A = \begin{bmatrix} 1 & -1 & 1 & -2 & 1 \\ 1 & -1 & 1 & -3 & 2 \\ 2 & -1 & 2 & -1 & 1 \\ 3 & -2 & 3 & -4 & 3 \end{bmatrix}$.

(a) Find rref(A).

(b) Find a basis for the kernel of A.

(c) What is the rank of A? What is the dimension of the kernel of A?

(d) State the Rank-Nullity Theorem. Verify it in the case of A.

34. Let V be the vector space of all 2×2 matrices. Prove or disprove the following statements:

(a) The set $S_2 = \{A \in V; \det(A) = 0\}$ is a subspace of V.

(b) The transformation $L : V \to V$ defined by $L(A) = A^T$ is a linear transformation.

35. Prove which of the following are subspaces under matrix addition and which are not. For those that are subspaces, find a basis.

    (a) The set of invertible 2×2 matrices.

    (b) The set of 2×2 matrices with 0 in the upper right corner.

    (c) The set of 2×2 matrices with determinant 1.

36. Let
$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}.$$
Find an orthonormal basis for the space spanned by $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_3$. Find the projection of $\vec{v} = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$ onto this space.

37. Let S be the subspace of $\mathbb{R}^3$ given by $x+y+z=0$. Verify that $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ form a basis of S.

38. True or False:
Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation. If $\ker T = \{0\}$, then the image of T is all of $\mathbb{R}^2$.

39. For each of the following sets of vectors, find out whether they are linearly independent or linearly dependent:

    (a) $\vec{v}_1 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} -1 \\ 3 \\ 8 \end{bmatrix};$

    (b) $f_1 = e^x$, $f_2 = e^{3x}$, $f_3 = e^{-2x}$;

(c) $g_1 = \sin^2 x,\ g_2 = \cos^2 x,\ g_3 = 1;$

(d) $\vec{w}_1 = \begin{bmatrix} 4 \\ 7 \\ 3 \\ 1 \\ 2 \end{bmatrix}, \vec{w}_2 = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 8 \\ -2 \end{bmatrix}, \vec{w}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 7 \\ 1 \end{bmatrix}, \vec{w}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 6 \end{bmatrix};$

(e) $\vec{u}_1 = \begin{bmatrix} 3 \\ 8 \\ 11 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} -8 \\ 4 \\ 10 \end{bmatrix}, \vec{u}_3 = \begin{bmatrix} 7 \\ -2 \\ 18 \end{bmatrix}, \vec{u}_4 = \begin{bmatrix} 6 \\ -8 \\ 19 \end{bmatrix}.$

40. True or false?

(a) The intersection of two linear subspaces is always a subspace.

(b) The union of two linear subspaces is always a subspace.

41. Complete each of the following sentences with the *definition* of the italized word or phrase:

(a) The vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ are *dependent* if ...

(b) The vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ form a *basis* for the linear space V if ...

(c) Let V and W be linear spaces. A function $T : V \to W$ is a *linear transformation* if ...

42. Which of the following sets of vectors are linearly independent?

(a) $S_1 = \{(-4, 1, 3), (1, 2, 5), (-3, 3, 8)\}$.

(b) $S_2 = \{1 + 2x + x^2, 1 + 4x - x^2, 1 + x + 2x^2\}$.

(c) $S_3 = \left\{ \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} \right\}.$

43. Let $L : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by

$$L \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 + x_3 \\ x_1 - x_3 \\ -x_1 + 3x_2 + 4x_3 \\ x_1 + 3x_2 + 2x_3 \end{bmatrix}.$$

(a) Find bases for the kernel and the image of L.

(b) State and verify the rank-nullity theorem for L. Find the rank of the matrix of L.

44. (a) Let $T : P_3 \to P_3$ be the linear transformation from the space of polynomials of degree at most 3 into itself given by

$$T(p(x)) = \frac{dp}{dx} + p(x).$$

Check whether T has an inverse and, if it does, find it.

45. If A is a 64×17 matrix of rank 11, how many linearly independent vectors $\vec{x}$ satisfy $A\vec{x} = 0$? How many satisfy $A^T\vec{x} = 0$? Justify.

46. Are the matrices

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 6 & 7 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

similar or not? Justify.

47. In the space P_2 of polynomials of degree at most 2 define the transformation T by $T(p) = 3p - 2p'$. Find the matrix of T with respect to the standard basis $1, t, t^2$.