

# Solutions to Review Multiple Choice

No hamsters were harmed in any way as a result of Math 317. Please don't tell the dean about the hamsters.

## Question 1 - Answer B

You can find pictures of such things by internet search.

## Question 2 - Answer B

One suggestion made by a student was to consider only one hole at a time. So first, imagine the wheel is open at  $y = 4$  (boundary  $C_1$ ) but closed off at  $y = 9$ . If we were to smoothly collapse the can down to a disk with boundary  $C_1$ , the “inward” normals would end up pointing in the direction of *decreasing*  $y$ .

Using the right-hand rule, this means that to use Stokes' Theorem,  $C_1$  must be oriented counter-clockwise as viewed from the origin (or the *negative*  $y$ -axis), or clockwise as viewed from the positive  $y$  axis (i.e. clockwise if viewed from far out looking down the positive  $y$  axis).

The parametrisation given for  $C_1$  in the problem has the usual counter-clockwise orientation in the  $xz$  plane viewed the “usual way”. By “usual way” I mean  $x$ -axis extending positively to the right and  $z$ -axis extending positively upward. However, to see the  $xz$  plane this way, you have to be looking from the *negative*  $y$ -axis toward the origin.

Taking the last two paragraphs together, we see that  $C_1$  is already oriented correctly for Stokes' Theorem.

Analysing  $C_2$  is similar. Note: This question was intentionally made as complicated as I could manage! If you can understand this one, you're all set.

## Question 3 - Answer C

Review the domain and range of grad, div and curl in 3D. Grad takes scalar functions to vector fields, curl takes vector fields to vector fields, and div takes vector fields to scalar fields. So you can't compose div with itself.

See the handout from Dec 1st.

## Question 4 - Answer E

Flux through a closed (i.e. boundaryless) surface is zero in curl fields, i.e. in fields which are the curl of another field. (To see this, apply either Stokes' Theorem or the Divergence Theorem to  $\iint_{\partial E} \text{curl } \mathbf{F} \cdot d\mathbf{S}$ .) To test if something is a curl field, check if its divergence is zero. (Note: all these fields are defined on all of  $\mathbb{R}^3$ , which is simply connected, so this test is reliable.) Checking the divergence of each of these fields, we see that both B and C have zero divergence, but A does not.

## Question 5 - Answer D

These fields are all defined on all of  $\mathbb{R}^2$ , which is simply connected, so we can see which are conservative by computing the 2D curl,  $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$ .

## Question 6 - Answer D (I especially like this one for understanding)

The divergence theorem states that the total outward flux through the hamster cage must be equal to the divergence within it. Divergence is  $(1g/hr)/m^3$  over a volume of  $1m^3$ , so the total divergence is  $1g/hr$  inside the cage. Therefore

$$(\text{flux through top}) + 4 \cdot (\text{flux through each side}) + (\text{flux through floor}) = 1g/hr$$

The problem states the outward flux through the floor is 0, and through each of the four sides the outward flux is  $-1g/hr$ . Hence the outward flux through the top is 5.

Intuitively, we say that 1 unit of stuff is being “created” inside the cage, and 4 units are rushing in. To balance the books, 5 units must be rushing out the top.

## Question 7 - Answer A

The tangent plane is intrinsic to the surface itself – without knowing a parametrisation, you can imagine “fitting” a tangent plane against the surface. However, each tangent plane has two choices of unit normal, and this choice is determined by the choice of parametrisation, since we calculate it from the parametrisation as  $\frac{\mathbf{r}_u \times \mathbf{r}_v}{|\mathbf{r}_u \times \mathbf{r}_v|}$ . The orientation of the surface is the same as giving the normal, so it also depends on the parametrisation. The grid curves definitely depend on the parametrisation (see the extra problem in Homework 10 for more on that). And finally, I believe some parametrisations just make people happier than others.

## Question 8 - Answer B

I suggest you work on this and graph them using a graphing program to check. The reason  $B$  is different than  $A$  is that as the spiral rises, it starts off in a different direction. Part  $C$  just changes  $t$  for  $t - \pi$  and then disguises things using the trig identity  $\cos(t - \pi) = -\cos(t)$  (similarly for  $\sin$ ). Another trig identity will show that  $D$  is the same as  $A$ . And finally,  $E$  is trading out  $t$  for  $t^3$  but these are both increasing from 0 to 1 so the curve is the same even though the “particle travels at a different speed.”

## Question 9 - Answer E

I made this one so tricky it took my husband at least 5 minutes of staring at it to figure it out! It’s because he was convinced they were all the same, and couldn’t see any one that was different. In fact, they are all the same *except for the domains*. Graph  $E$  and  $A$  to see what I mean about the domains – the boundary of  $A$  is a circle, while the boundary of  $E$  is a distorted square.

Note: It may appear that  $A$  and  $B$  have circles going around in different directions. In fact they don’t, but even if they did (if it were  $\sin$  and  $\cos$  instead of  $\sin$  and  $-\cos$ ), this still wouldn’t change the surface!

## Question 10 - Answer E

This question is to remind you of the divergence form of Green's Theorem. It says that the flux through the circle is equal to the integral of divergence inside, i.e.

$$\int_C \mathbf{F} \cdot \mathbf{n} ds = \iiint_D \operatorname{div} \mathbf{F} dV$$

(It's a 2D form of the Divergence Theorem.) The divergence of both B and E are zero, and the others are non-zero. (Note:  $\operatorname{div} \mathbf{F}$  being non-zero doesn't guarantee the integral of  $\operatorname{div} \mathbf{F}$  over some region is non-zero; so in a time-pressure situation – say, a life-and-death vector calculus duel – you would mark A, C and D as “unlikely” and only come back to them if you didn't find an answer to the problem among the remaining options B and E.)

So we seem to have two possible correct answers: B and E. However, Green's Theorem (in either circulation/curl or flux/divergence form) doesn't apply to B, since the field is not defined at the origin (inside the unit circle). A calculation (or visualising the field B) will convince you the flux is non-zero for B.

So the answer is E.

Note: The field B is very interesting. It's been a special example to us in many situations.

## Question 11 - Answer A

For practice, you can parametrise it yourself. As a multiple choice question, you can just verify whether each of the given parametrisations satisfies the equations. This is to remind you to check your work. Supposing you got answer B, you would then check if your answer satisfies  $z = x^2$  and find the error in your work. In this way you will discover that none of B, C, D or E is correct.

The correct answer (A) is disguised by the trig identity  $1 = \cos^2 t + \sin^2 t$  (just to make it more fun).

## Question 12 - Answer C

You can write this out and solve it using standard methods, but that takes a while. Here's my way of thinking about it (which will be good practice for your visual intuition):

Consider the upward component of velocity. This is 3 in part A, so let's keep using part A for this example. Upward velocity decreases at a rate of 1 (since units aren't given, let's suppose speed is  $m/s$  and acceleration is  $m/s^2$  for aid in visualising). So after exactly  $3s$ , upward velocity stops – this is the top of the projectile's arc, midway through its journey. Hence in  $6s$ , it hits its target. The target is  $1m$  away, so it must have a horizontal velocity (imagine the movement of its shadow on the ground) of  $(1/6) m/s$  to be in the right place at the right time. Hence indeed, initial velocity A will result in a smack by the hamster bomb.

Doing this sort of check on each answer will reveal answer B.

## Question 13 - Answer D

All of these vector fields are defined on all of  $\mathbb{R}^3$ , so we can check if they are the curl of another vector field by checking if their divergence is zero. The only one with non-zero divergence is D.