Homework 8 Extra Problems - Solutions

Problem 1

There are lots and lots of possible solutions. For some great extra practice, draw pictures of all the regions I give as example solutions here.

Write down an example of a region that is:

1) not connected

Some possible examples:

$$\{(x,y) : x \neq 1\}, \quad \{(x,y) : x^2 + y^2 < 1 \text{ or } x^2 + y^2 > 2\},$$

$$\{(x,y) : x^2 + y^2 \neq 1\}, \quad \{(x,y) : x \neq 1 \text{ and } y \neq 2\}.$$

2) connected but not simply connected

Some possible examples:

$$\{(x,y) : (x,y) \neq (1,1)\}, \quad \{(x,y) : x^2 > 1 \text{ and } y^2 > 1\},$$

$$\{(x,y) : \cos(x) \neq 0 \text{ or } \cos(y) \neq 0\}, \quad \{(x,y) : x \neq 1 \text{ or } y \neq 2\}.$$

3) simply connected

Some possible examples:

$$\{(x,y) : x < 1\}, \quad \{(x,y) : x^2 < 1 \text{ and } y^2 < 1\},$$

$$\{(x,y) : x^2 \geq 0\}, \quad \{(x,y) : x < y^2\}.$$

Problem 2

Problem: One consequence of Kepler's Second Law is that a planet in an elliptical orbit moves faster while it is closer to the sun. Explain why this is a consequence of conservation of energy (qualitatively)

Answer: First, it is important to point out that a planet orbiting a sun is moving under the influence of the gravitational force field and no other forces. In this case, conservation of energy applies. For the gravitational field created by the sun, the potential energy of the planet is a function of the distance to the sun. If it is closer to the sun, the potential energy is lower. If it is farther from the sun, the potential energy is higher. By conservation of energy, if the planet is closer to the sun, then it has lower potential energy and therefore higher kinetic energy than when it is farther from the sun. But higher kinetic energy means higher speed. So a planet goes faster when it is closer to the sun.